Name: _____

Exam # 1 – Math 2311H – Spring 2014

Directions: remember to show work in the free response part.

1. For which of the following can you definitively say that f is continuous at $x = 4$.

   (A) $\lim_{x\to 4} f(x)$ exists.

   (B) $\lim_{x\to 4^+} f(x)$ exists.

   (C) f is defined on an open interval that contains $x = 4$.

   (D) $\lim_{x\to 4} f(x) = f(4)$.

   (E) none of the above.

2. $$f(x) = \begin{cases} x^2 + 1 & -1 \le t < 1 \\ -x + 1 & 1 \le x < 2 \\ -1 & x > 2 \end{cases}$$

   The points in the domain of f at which f is continuous are

   (A) $(-1, 1) \cup (1, \infty)$ (B) $[-1, 1) \cup (1, \infty)$ (C) $(-1, 1) \cup (1, 2) \cup (2, \infty)$

   (D) $[-1, 1) \cup (1, 2) \cup (2, \infty)$ (E) $(-1, 2) \cup (2, \infty)$

3. $\lim_{x\to \frac{\pi}{2}} \dfrac{\sin x}{x}$

   (A) 1 (B) 0 (C) $\dfrac{\pi}{2}$ (D) $\dfrac{2}{\pi}$ (E) Does not exist

4. $$f(x) = \begin{cases} x^2 - 6x + 13 & \text{for } x \ge 3; \\ kx + 1 & \text{for } x < 3. \end{cases}$$

   The function f is defined above. For what value of k, if any, is f continuous at $x = 3$.

   (A) 1 (B) 2 (C) 3 (D) 7 (E) No value of k.

5. What is $\tan(\frac{\pi}{6})$?

(A) 0 (B) $\frac{\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) $\frac{1}{2}$ (E) 1

6. Compute $\lim_{x\to 2} \dfrac{x^3-8}{x^2-4}$.

(A) 4 (B) 0 (C) 1 (D) 3 (E) DNE

7. Which of the following statements about the function

$$f(x) = \begin{cases} 2x & 0 < x < 1; \\ 1 & x = 1; \\ -x+3 & 1 < x < 2. \end{cases}$$

is not true?

(A) $f(1)$ does not exist.

(B) $\lim_{x\to 0^+} f(x)$ exists.

(C) $\lim_{x\to 2^-} f(x)$ exists.

(D) $\lim_{x\to 1} f(x)$ exists.

(E) $\lim_{x\to 1} f(x) = f(1)$.

8. Compute $\lim_{x\to 1^+} \dfrac{x^2+2x+1}{x-1}$.

(A) $+\infty$ (B) $-\infty$ (C) 0 (D) 1 (E) -1

9. Suppose $\lim_{x\to -3^-} f(x) = -1$, $\lim_{x\to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statements are true?

I. $\lim_{x\to -3} f(x) = -1$

II. $f(x)$ is continuous at $x = -3$.

III. $f(x)$ has a removable discontinuity at $x = -3$

(A) I. only (B) I. and III. only (C) III. only (D) II. only (E) I., II., and III.

10. $\lim_{x\to 0} \dfrac{\sin 5x}{x} =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) DNE

Extra Credit:

11. Let

$$f(x) = \begin{cases} \dfrac{3x(x-1)}{x^2-3x+2} & \text{for } x \neq 1, 2; \\ -3 & \text{for } x = 1 \\ 4 & \text{for } x = 2 \end{cases}$$

Then $f(x)$ is continuous

(A) at all real numbers except at $x = 1$

(B) at all real numbers except at $x = 2$

(C) at all real numbers except at $x = 1, 2$

(D) at all real numbers except at $x = 0, 1, 2$

(E) at each real number

12. $\lim_{x\to 0} x \sin \dfrac{1}{x} =$

(A) 0 (B) ∞ (C) $-\infty$ (D) 1 (E) $\sin 1$

13. Evaluate the limit analytically (use algebra). Show work.

$$\lim_{x \to 0} \frac{\sqrt{5+x} - \sqrt{5}}{x}$$

14. Consider the function $f(x) = \dfrac{x^2 + x}{x - 1}$ over the interval $[\frac{5}{2}, 4]$. Compute $f(\frac{5}{2})$ and $f(4)$. What does the Intermediate Value Theorem say about $f(x)$ and whether there is a $c \in [\frac{5}{2}, 4]$ such that $f(c) = 6$. Find such a c.

15. State the definition for the following symbol

$$\lim_{x \to a} f(x) = L.$$

16. Evaluate the limit analytically (use algebra). Show work.

$$\lim_{\theta \to 0} \frac{1 - \cos\theta}{2\sin^2\theta}.$$

Extra Credit

17. Explain what the following terms mean: removable discontinuity, jump discontinuity, vertical asymptote, continuous.