Name: _____

Exam # 1 – Math 2311H – Fall 2013

Directions:

1.

$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \geq 2; \\ kx + 1 & \text{for } x \geq 2. \end{cases}$$

The function f is defined above. For what value of k, if any, is f continuous at $x = 2$.

(A) 1 (B) 2 (C) 3 (D) 7 (E) No value of k.

2. The graph of the even function $y = f(x)$ consists of line segments as shown above. Which of the following statements about $f(x)$ is false.

(A) $\lim_{x \to 0}(f(x) - f(0)) = 0$

(B) $\lim_{x \to 0} \dfrac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x \to 0} \dfrac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x \to 2} \dfrac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$ does not exist.

3. Which of the following statements must be true.

   I. If f is a continuous function on a closed interval $[a, b]$, then there is some $c \in [a, b]$ such that $f(c) = 0$.

   II. If f is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

   III. If f is continuous at $x = 0$, then $\lim_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$ exists.

   (A) I. only (B) II. only (C) II. and III. only (D) I., II., and III. (E) I. and III.

4. What is $\cos(\frac{\pi}{6})$?

   (A) 0 (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$ (E) 1

5. Compute $\lim_{x \to 4} \dfrac{t^2 - 16}{t - 4}$.

   (A) $+\infty$ (B) 4 (C) 0 (D) 8 (E) DNE

6. Compute $\lim_{x \to 3^-} \dfrac{|x - 3|}{x - 3}$.

   (A) $+\infty$ (B) $-\infty$ (C) 0 (D) 1 (E) -1

7. Compute $\lim_{x \to 1^+} \dfrac{x^2 + 2x + 1}{x - 1}$.

   (A) $+\infty$ (B) $-\infty$ (C) 0 (D) 1 (E) -1

8. Let $f(x) = \frac{x^2 - 4}{|x - 2|}$. Which of the following limits exist?

   I. $\lim_{x \to 2^-} f(x)$

   II. $\lim_{x \to 2^+} f(x)$

   III. I. $\lim_{x \to 2} f(x)$

   (A) I. only (B) II. only (C) I. and II. only (D) I., II., and III. (E) none

9. If $f(x) = \frac{1}{2+10^{\frac{1}{x}}}$, then $\lim_{x\to 0} f(x) =$

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) DNE

10. Let

$$f(x) = \begin{cases} \dfrac{3x(x-1)}{x^2-3x+2} & \text{for } x \neq 1, 2; \\ -3 & \text{for } x = 1 \\ 4 & \text{for } x = 2 \end{cases}$$

Then $f(x)$ is continuous

(A) at all real numbers except at $x = 1$
(B) at all real numbers except at $x = 2$
(C) at all real numbers except at $x = 1, 2$
(D) at all real numbers except at $x = 0, 1, 2$
(E) at each real number

11. Let

$$f(x) = \begin{cases} \dfrac{x^2-x}{2x} & \text{if } x \neq 1; \\ 4 & \text{for } x = 1 \end{cases}$$

Which of the following statements are true?

I. $\lim_{x\to 1} f(x)$ exists.
II. $f(1)$ exists.
III. f is continuous at $x = 1$

(A) I. only (B) II. only (C) I. and II. only (D) none (E) I., II., and III.

12. $\lim_{x\to 0} \dfrac{\sin 5x}{x} =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) DNE

13. $\lim_{x\to 0} \dfrac{x^2-4}{x^2+4} =$

(A) 1 (B) 0 (C) $\frac{-1}{2}$ (D) -1 (E) ∞

14. $\lim_{x\to 0} x \sin\frac{1}{x} =$

(A) 0 (B) ∞ (C) $-\infty$ (D) 1 (E) $\sin 1$

For questions 15. thru 18. use the following function and graph:

$$f(x) = \begin{cases} 1-x & \text{if } x \in [-1,0) \\ 2x^2-2 & \text{if } x \in [0,1] \\ -x+2 & \text{if } x \in (1,2) \\ 1 & \text{if } x = 2 \\ 2x-4 & \text{if } x \in (2,3] \end{cases}$$

15. $\lim_{x\to 2} f(x) =$

(A) 0 (B) 1 (C) 2 (D) DNE (E) -1

16. The function f has a removable discontinuity at $x =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) no removable discontinuities

17. On which of the following intervals is f continuous?

(A) $[-1,0]$ (B) $(0,1)$ (C) $[1,2]$ (D) $[2,3]$ (E) none of these

18. The function f has a jump discontinuity at $x =$

(A) -1 (B) 1 (C) 2 (D) 3 (E) none of these

19. Use algebra to evaluate the limit. Show work.

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$$

20. Use an appropriate trig identity learned in class to compute the following limit. I want to see work; not just the calculator.

$$\lim_{x \to 0} \frac{1 - \cos x}{\sin x}$$

21. Let $f(x) = \sqrt{x(x-1)}$. Be sure to show work.

    (a) Find the domain of $f(x)$.

    (b) Find $\lim_{x \to 0^-} f(x)$

    (c) Find $\lim_{x \to 1^+} f(x)$

22. State the definition for the following symbol

$$\lim_{x \to a} f(x) = L.$$

23. Let $f(x) = x^2 - 5$.

    (a) Find the derivative of $f(x)$ by the limit process.

    (b) Find the equation of the line that is tangent to the graph of f at the point $(3, 4)$.

[Bonus.] The function f is given by $f(x) = \frac{ax^2+12}{x^2+b}$. The figure below shows a portion of the graph of f. which of the following could be the values of the constants a and b?

(A) $a = -3, b = 2$

(B) $a = 2, b = -3$

(C) $a = 2, b = -2$

(D) $a = 3, b = -4$

(E) $a = 3, b = 4$