Conference on Ordered Algebraic Structures

May 9th - 11th, 2025
Wilkes Honors College
Florida Atlantic University
Jupiter, Fl

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- The Wilkes Honors College of Florida Atlantic University
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A subframe of Polars of an M-frame

Papiya Bhattacharjee

Abstract

Given an M-frame L, that is, algebraic satisfying the FIP, we denote the collection of all compact elements of L by $\mathfrak{K}L$. Let L' be a subframe of L where each $x \in L'$ is a join of polars of $\mathfrak{K}L$. The frame L' is crucial in the study of $\text{Min}(L)^{-1}$, the space of minimal prime elements of L with respect to the inverse topology. We will discuss different properties of L', in relation to the properties of L. We will also explore how the topological properties of $\text{Min}(L)^{-1}$ depend on the frame theoretic properties of L'. For example, we will show that when $\text{Min}(L)^{-1}$ is a discrete space, the Boolean algebra of its clopen sets, $\text{Clop}(\text{Min}(L)^{-1})$, is isomorphic to $\mathcal{P}(L')$.

Suprema Functions

Ricardo Carrera

Abstract

For a frame F, $\mathcal{R}F$ is the ring of continuous functions on F. In this talk we present some preliminary results about the existence and properties of supremum of functions in $\mathcal{R}F$. In particular, for $F \in \text{KReg}$, the category of compact regular frames and frame homomorphisms, we provide the frame-theoretic counterpart of certain classical results about the supremum of functions and discuss how this work is related to the quasi-F hull of a compact regular frame F. Lastly, if time permits, we discuss some of the broader applications of this work.

Topological representations of distributive lattices

Christian Corbett

Abstract

The interplay between topology and order is a central theme in the study of distributive lattices. Priestley duality sets up a functorial bridge from algebra to topology, revealing how lattice-theoretic properties manifest in the behavior of ordered topological spaces. We examine how certain lattice and topology characterizations reflect across this duality, and explore examples drawn from particular classes of lattices that illustrate the power of the approach.

Continuous Algebra: Algebraic Semantics for Continuous Propositional Logic

Purbita Jana, joint work with Prateek Keatra

Abstract

Continuous logic extends classical logic to systems with infinitely varying truth values, enabling the study of graded truth systems. This work introduces continuous algebra as the algebraic semantics for Continuous Propositional Logic (CPL), a generalization of Łukasiewicz logic with added operators and axioms for continuity. Continuous algebras extend MV-algebras by incorporating a unary operator κ, corresponding to the unary connective $\frac{1}{2}$ in CPL. We establish structural results, including the subdirect representation theorem, and introduce lu^*-groups to connect continuous algebras with algebraic representations. Leveraging these results, we prove the weak completeness of CPL, providing a robust algebraic framework for its study.

The Prime Spectrum of an Artinian Frame

Ramiro Lafuente-Rodriguez

Abstract

A frame is said to be *Artinian* if it satisfies the descending chain condition. We examine the construction of the prime spectrum of an algebraic frame and investigate the conditions under which this spectrum forms an Alexandrov space. We equip the spectrum with the specialization order and interpret it as a lattice. Since the specialization order reflects the order structure of the underlying Artinian frame, we argue that the resulting topological space inherits the Alexandrov property. We further propose the following conjecture: if X is a T_0-topological space whose specialization order $\leq_A$ turns $(X, \leq_A)$ into an Artinian frame, then X is an Alexandrov space.

An Overview of the Pierce-Birkhoff Conjecture

James Madden

Abstract

The conjecture (dating from 1956) is that every (finitely) piecewise polynomial function from $\mathbb{R}^n$ to $\mathbb{R}$ is a (finite) max of (finite) mins of polynomials. In 1983, L. Mahé [5] proved the conjecture for $n = 2$, but it remains unresolved for $n > 2$. In 2010, Sven Wagner [6] proved the generalization of the PBC to smooth surfaces using the theory of separating ideals introduced in [4] and [1]. The latter reference is based on Zariski's theory of complete ideals in 2-dimensional local rings [7]. Numerous works of Mark Spivakovsky and coauthors attempt to find similar methods that may work in higher dimensions; see, e.g., [2]. Achieving this may involve developing generalizations of Zariski's theory to higher dimensions—a project that Zariski himself expressed interest in, but about which little is known (but see [3]). Proving the PBC (or finding a counterexample) involves understanding the set of polynomials that change sign between two points of the real spectrum of the ring of real polynomials in 3 or more variables. In this talk, I will describe attempts to construct interesting examples, and I will relate how my work on the conjecture has led me to topics in combinatorial commutative algebra, semigroup cohomology, and ordered rings.

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Super-algebraic frames and restricted Hahn Groups

Warren Wm. McGovern

Abstract

Let L be a frame. An element $c \in L$ is said to be *super compact* if whenever $c \leq \bigvee x_i$, then $c \leq x_i$ for some $i \in I$. The frame L is said to be *super-algebraic* if every element is the join of super compact elements. We shall discuss super algebraic frames and their relationship to restricted Hahn groups over partially-ordered sets.

On pointfree notions of generalized compactness notions

Inderasan Naidoo, joint work with Mark Sioen

Abstract

We introduce and study some new covering properties in frames that are pointfree variants of generalized compactness notions. We establish the concept of star-finite covers on a frame and introduce Hypocompact frames. (Co)refecting such frames via nearness completion [1] is proposed. Variants of point-finite covers ([2, 3, 4]) and associated variants of pointfree metacompactness are studied. The talk is an exposition of these new concepts and work in progress.

References

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Local real closed SV-rings of finite rank and their model theory

Ricardo Jesús Palomino Piepenborn

Abstract

A commutative unital ring is an SV-ring if all its residue domains (that is, all its quotients by prime ideals) are valuation rings. SV-rings were first introduced in the context of rings of continuous functions C(X), and later studied in the more general context of f-rings. Among all f-rings, Schwartz's real closed rings form a subclass which play a central role in semi-algebraic geometry; examples of such rings are the rings C(X) and rings of continuous semi-algebraic functions on a semi-algebraic set over a real closed field. Residue domains of real closed SV-rings are real closed valuation rings (equivalently, convex subrings of real closed fields), and their model theory is well understood. The goal of this talk is to present an approach to the model-theoretic analysis of some classes of local real closed SV-rings in terms of their residue domains.

A local real closed SV-ring has rank n if it has exactly n minimal prime ideals, and the class of local real closed SV-rings of rank n is elementary in the language of rings; a geometric example of such a ring is the ring of germs G of continuous semi-algebraic functions at a point in a semi-algebraic curve with exactly n half-branches. First, I will spell out a representation theorem for local real closed SV-rings of rank n, which describes them as iterated fibre products of n non-trivial real closed valuation rings (that is, those real closed valuation rings which are not fields). I will then focus on the class of n-fold fibre products of non-trivial real closed valuation rings with isomorphic residue field along their residue field map; an example of such a ring is the ring of germs G. I will sketch proofs of the main model-theoretic properties of this class, in particular:

1. It has a recursive axiomatization in the language of rings and its theory is complete (hence decidable); and

2. Its theory is model complete and it is the model companion of the theory of local real closed (SV-) rings of rank n.

The same results as in 1) hold for another class of n-fold fibre products of non-trivial real closed valuation rings. As a consequence, the theory of local real closed SV-rings of rank 2 has exactly two completions, both of which are decidable. I will conclude by showing how these results suggest an elementary classification of all local real closed SV-rings of finite rank in terms of the isomorphism type of their poset of branching prime ideals.

Model-theoretic properties of simple dimension groups
Philip Scowcroft

Abstract

A dimension group is a partially ordered Abelian group whose partial order is directed and isolated and has the Riesz interpolation property. A dimension group is simple just in case it has no nontrivial ideals: i.e., just in case every strictly positive element is an order unit. After surveying special properties of existentially closed simple dimension groups, this talk will focus on positive formulas to describe the injective simple dimension groups.

Some algebraic characterizations of U-frames
Batsile Tlharesakgosi

Abstract

A completely regular frame L is a *U-frame* whenever u and v are cozero elements in L such that $u \wedge v = 0$, then there exists a complemented element $c \in L$ such that $u \leq c$ and $c \wedge v = 0$. In this talk, we will give algebraic characterizations of U-frames in terms of ring-theoretic properties of the ring $\mathcal{R}L$ of real-valued continuous functions on a completely regular frame L. We show that a frame is a U-frame if and only if it is an F-frame and its Čech-Stone compactification is zero-dimensional. If L and M are compact frames, and one of them is zero-dimensional such that $L \oplus M$ is a U-frame, then each summand is a U-frame.

Covering Relations in the Poset of Combinatorial Neural Codes
Matthew Trang

Abstract

A combinatorial neural code $\mathcal{C}$ is a subset of $2^{[n]}$ for some $n \in \mathbb{N}$ where $[n] = \{1, \ldots, n\}$. Each $i \in [n]$ represents a neuron, and each element $\sigma \in \mathcal{C}$ represents a codeword which records the co-firing event of some neurons. Consider a space $X \subseteq \mathbb{R}^d$, simulating an animal's environment, and a collection $\mathcal{U} = \{U_1, \ldots, U_n\}$ of open subsets of X. Each $U_i \subseteq X$ simulates a receptive field corresponding to a specific region where neuron i is active. Then, the code of $\mathcal{U}$ in X is defined as $\text{code}(\mathcal{U}, X) = \left\{ \sigma \subseteq [n] \,\middle|\, \bigcap_{i \in \sigma} U_i \setminus \bigcup_{j \notin \sigma} U_j \neq \emptyset \right\}$. If a neural code $\mathcal{C} = \text{code}(\mathcal{U}, X)$ for some X and $\mathcal{U}$, we say $\mathcal{C}$ has a realization. Moreover, if $\mathcal{U}$ consists of open convex subsets of X, we say $\mathcal{C}$ is a convex neural code; otherwise, it is non-convex. A prior study constructed a poset of distinct isomorphism classes of neural codes, partially ordered by the relation "is a minor of" and denoted by $\mathbf{P_{Code}}$. A recent conjecture posits that every convex neural code can be realized as a minor of a neural code arising from a representable oriented matroid. While this conjecture has been confirmed in dimension two, its validity in higher dimensions remains an open question. To advance the investigation of this conjecture, we focus on the combinatorial structure of the poset $\mathbf{P_{Code}}$. In this work, we provide a complete characterization of the covering relations within $\mathbf{P_{Code}}$, along with an algorithm for enumerating all distinct isomorphism classes of covering codes associated with a given neural code.

Remarks on semilattices of compact open sets

Marcus Tressl

Abstract

The representation theory of a semilattice S asks about how to "encode" S in some topological space. There are two approaches: the first one represents S as a subspace of a space (Pontryagin like approach); the second one represents S as set of open sets of a space (Stone like approach). While the former has fairly complete answer(s), the latter is under investigation ever since and this talk is about that second approach, where the join semilattice $\mathrm{KO}(X)$ of compact and open sets of some space X plays a central role: Classical Stone representation says that every distributive lattice is of the form $\mathrm{KO}(X)$ for some space X and Grätzer later extended this to distributive join semilattices. For a while I believed that $\mathrm{KO}(X)$ might be distributive for any space, but this turned out to be outright false. In the talk I will show some counterexamples and comment on possible further directions by exhibiting a topological framework to simultaneously address both approaches of the representation problem above.

Existentially closed prime-model extensions of Archimedean lattice-ordered groups with strong unit

Brian Wynne

Abstract

Let $\mathbf{W}^+$ be the category of non-zero Archimedean ℓ-groups with distinguished strong unit and unit-preserving ℓ-group homomorphisms. If G is a $\mathbf{W}^+$-object, then G is existentially closed (e.c.) in $\mathbf{W}^+$ if any finite system of ℓ-group equations and inequations, involving elements of G, that is solvable in some extension of G in $\mathbf{W}^+$ is solvable already in G. In [1], among other things, Scowcroft establishes axioms for the e.c. $\mathbf{W}^+$-objects, shows that $C(\beta\mathbb{N} \setminus \mathbb{N})$ is e.c. in $\mathbf{W}^+$, and gives many other examples. Building on Scowcroft's work, in [2] I give a different set of axioms for the e.c. $\mathbf{W}^+$-objects and use them to characterize those $C(X)$ that are e.c. in $\mathbf{W}^+$. Now e.c. $\mathbf{W}^+$-objects are to $\mathbf{W}^+$-objects as algebraically closed fields are to fields, so a natural question is: To what extent is there is an analogue of algebraic closure for $\mathbf{W}^+$-objects? In model-theoretic terminology, the problem is to determine which $\mathbf{W}^+$-objects have e.c. prime-model extensions, where G' is an e.c. prime-model extension of G if G' is e.c. in $\mathbf{W}^+$ and whenever H is an e.c. extension of G in $\mathbf{W}^+$, then G' embeds into H over G. After giving background information on $\mathbf{W}^+$, and on e.c. $\mathbf{W}^+$-objects, I will discuss what I know so far about the existence and uniqueness of e.c. prime-model extensions in $\mathbf{W}^+$.

References

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