A Theory of Worker Turnover in R&D Intensive Firms

Kameshwari Shankar †
City College of New York

Suman Ghosh‡
Florida Atlantic University

February 12, 2013

Abstract

This paper builds a theoretical model to address evidence on labor mobility patterns in technology-intensive firms engaged in R&D. Labor turnover in these firms is characteristically different from turnover in traditional industries both in size and composition. Specifically, the pool of workers switching employers comprises of relatively productive workers. Our model explains these characteristics of labor turnover by focusing on the distinguishing features of R&D-intensive firms, in particular, the stochastic nature of returns to R&D investment and the transmission of R&D knowledge through worker movement, to explain patterns of labor mobility in these firms. The analysis provides implications of such labor mobility on industry growth.

1 Introduction

The organization of production and employment relationships in some modern industries, especially those involved in the research and development (R&D henceforth) of new products and processes, is characteristically different from traditional industries studied in the labor economics literature. There is a growing body of research focusing on labor markets
of high-technology industries that points to significant differences in employment relationships in these markets. For example, studies in the organizational sciences have documented that highly productive workers in high-technology markets switch employment often during the lifetime of their career and that worker turnover is perceived as a positive signal by employers. There appears to be no stigma attached to a worker moving from one firm to another. To the contrary, workers who change jobs frequently are seen as enterprising and talented, while employees who stay in the same firm risk acquiring a reputation as “dead wood”.¹ In the words of an engineer, “A man who has not changed companies is anxious to explain why; a man who has (changed companies) perhaps several times, feels no need to justify his actions.”² The presence of a positive signal in labor turnover is empirically evidenced by wage increases experienced by job changers. Morgan et al. (2004) find that job changers in IT occupations earned about 5 percent more than those who had not changed jobs. This is contrary to the predictions of standard adverse selection models where labor turnover, driven by asymmetric information about worker abilities, leads to a second-hand labor pool of low ability workers.³

Secondly, studies of engineers and technology workers have found that in many instances, turnover is not negatively correlated with tenure as most traditional theories would suggest.⁴ Studies of human resource practices in R&D intensive firms frequently find the presence of multiple ports of entry and a much reduced emphasis on performance incentives where employers select workers with longer tenures for career development and promotions.⁵ Thus, technology firms appear to be increasingly relying on the external labor market for filling not only entry level positions, but more senior level positions as well.

In order to develop a theory to explain the unique patterns of labor turnover described above, we build on the standard model of asymmetric employer learning with job assignments while incorporating some important characteristics of high-technology industries.⁶

²Saxenian (1994).
³Greenwald (1986).
⁴Josefek and Kauffman (2003); Joseph et al. (2007).
⁵Andersson et al. (2008).
⁶See Waldman (1984) and Ricart i Costa (1988) for standard models of asymmetric learning with job
As pointed out by Fallick et. al. (2006), high-technology firms in Silicon Valley are characterized by two distinguishing features. First, there is significant “job-hopping” which facilitates the reallocation of talent and resources toward firms with superior innovations and second, high-technology industries are characterized by innovations which are large but also uncertain. Further, as established by Moen (2005) the mobility of technical workers is a source of R&D knowledge spillover across firms. In this environment, worker turnover acts as a conduit for transferring R&D knowledge and hence can facilitate the efficient utilization of this knowledge across firms.

Theoretically, we introduce stochastic returns on investment in research and development and R&D knowledge accumulation transmitted across firms through worker mobility. The main argument we develop is the following. When workers are assigned to technically challenging R&D projects they acquire substantial access to critical R&D knowledge which they can then transmit to other firms if they switch employers in the future. Since output in these R&D projects is also likely to be more sensitive to the worker ability, higher ability workers will be assigned to such jobs. Stochastic returns on R&D implies that a firm, which had successful R&D returns in the past, may suffer a decline in its technology and hence may not currently have a good R&D project to utilize its workers in. The firm will then have a pool of highly productive workers who have accumulated R&D knowledge but do not have an opportunity to utilize it most productively there. The productivity of such workers will be higher in a new firm with a superior technology that has good investment returns this period. The resulting misallocation of worker productivities across firms will make some turnover of high ability workers efficient in our model. When such turnover is realized as the equilibrium outcome, we refer to it as a “complete turnover” of R&D workers in the second-hand labor market.

The model generates a number of interesting results that explain observed empirical facts. First, even with asymmetric information about worker productivities, the pool of workers changing jobs need not comprise low ability workers. In general, when the returns from R&D investment is highly variable and R&D knowledge is easily transferable across firms, we will see complete turnover of high ability R&D workers in the second-hand labor assignments.
market. Second, we are able to shed light on the relationship between R&D investment and worker turnover in firms where there is significant knowledge transfer through worker mobility. As shown by Moen (2005) in his study of Norwegian high-technology firms, we find that turnover is lower when R&D investment is higher. Third, we provide a framework for explaining why high-technology industries have moved away from traditional tenure-based promotion rules towards greater reliance on the external labor market even at higher levels of the job ladder. Finally, we also discuss the effect of worker mobility on the efficiency of investment and job assignments and we describe when and why labor mobility can enhance industry output. In particular we explain the often mentioned contrast between the success of high-technology firms in Silicon Valley and the decline of similar firms in Route 128 of Massachusetts in terms of differences in the extent of labor mobility found in these two regions. At the same time we caution that while labor mobility may have facilitated the success of high-technology firms in Silicon Valley, this need not be true for other industries where labor turnover is driven by fundamentally different industry characteristics.

The structure of the paper is as follows. In Section 3 we describe our contribution to the literature on labor turnover and knowledge transfer. Section 4 provides details of the theoretical model. Section 5 describes the equilibrium and efficient outcomes for job assignment, turnover and investment and the impact of turnover on industry output. Section 6 provides a general discussion of our results as they apply across other industries outside high-technology and then we conclude. All proofs are in the Appendix.

2 Related Literature

The literature in labor economics has typically addressed worker turnover in two ways. One stream of research has treated labor turnover as exogenous and explored patterns of wage, tenure and promotion as well as human capital formation in this framework. Much of this literature has used asymmetric learning in the labor market to predict adverse selection in the labor market. Thus when current employers have greater information about worker abilities than new employers, and labor turnover is exogenous, the second-
hand labor market will be composed of low ability workers only. There are at least two problems with applying these models to explain labor market outcomes in high-technology industries. First, since these studies assume exogenous worker turnover, they fail to explain the persistence of labor turnover in these industries. Second, while many empirical studies have confirmed the presence of asymmetric learning in labor markets in general and in the market for engineers in particular, there is strong evidence to suggest that the “marking effect” emphasized in the adverse selection literature, where a worker’s perceived ability decreases with each move to a new firm does not exist or is mitigated by other positive factors enabling mobility in high-technology labor markets. In contrast to these studies, our paper explains the persistence of worker turnover in high-technology endogenously. We argue that a worker’s productivity in a firm depends on the returns to R&D investment and volatility in those returns creates the possibility of turnover in equilibrium as workers move to firms with better investment returns where they can be more productive. Further, we draw a positive correlation between turnover and worker ability by incorporating job assignment decisions with respect to R&D-intensive jobs. We argue that, since higher productivity workers will be assigned to R&D-intensive jobs, volatility in R&D will have an impact on the retention of higher productivity workers when these workers can transfer R&D knowledge across firms through turnover. This generates a turnover pool of high ability workers contrasting the predictions of standard adverse selection models.

A second approach to labor turnover taken by the literature explains turnover decisions endogenously as we do in this paper. Existing economic models that explain endogenous labor turnover have used heterogeneous firm-worker matches and search-theoretic models of employment to explain observed turnover. However, neither of these models adequately explain the absence of a negative link between worker turnover and tenure found in many modern labor markets. In matching models, as the quality of the firm-worker match is

---

8 A recent paper by Kahn (2012) finds a stronger presence of asymmetric learning in occupations such as computer programmers, researchers and technicians than among workers in professional service industries such as consultants, lawyers, accountants, health-care givers etc. See also DeVaro and Waldman (2012).
9 Burdett (1978); Jovanovic (1979); Rogerson et. al. (2005); Mortensen and Pissarides (1999).
revealed over time, separations occur when the match quality is lower than the average expected quality in the labor market. But once the true match quality is revealed to be high, the possibility of turnover no longer exists for the worker. A key contribution of our model is that we introduce temporal variability in firm-worker match quality and we argue that the true firm-worker match quality itself might change over time depending on the stochastic realization of technology. Thus a worker may produce high output in the firm when it has a good return on R&D. But the match quality between the same firm and worker may deteriorate in the future if the firm’s technology declines. An important implication is that tenure and turnover are no longer negatively correlated. Unlike previous models of promotions and turnover that focused on internal labor markets, our model generates the possibility of new worker hires at higher job levels. Some search-theoretic models, such as the one described by Coles and Mortensen (2011), also allow stochastic productivity shocks at the firm-level to generate efficient turnover in a similar manner to ours. However, these models do not account for differences in worker productivity that influence the characteristics of turnover as we do in the current paper. This is important for two reasons. One, it allows us to draw a relationship between job assignments, human capital acquisition and turnover. Second, it also allows us to capture the impact of turnover on overall industry growth through its effect on the firm’s job assignment and investment decisions.

Our work is also closely related to the literature on labor pooling in industrial clusters. Combes and Duranton (2006) look at a model where labor pooling improves worker mobility between firms and allows for better firm-worker matches, but also heightens labor market competition through labor poaching. Similarly Gerlach et. al. (2009) consider the interaction between labor pooling, R&D investment and the firms’ location decisions. They establish a positive relationship between the riskiness of R&D investments, lessened labor market competition and an increased incentive to co-locate. As in these papers, we also consider labor market competition when workers transfer knowledge across firms through mobility and we also find greater labor mobility when investment returns are more volatile. However, we do not model the decision of firms to cluster, but instead focus on the patterns

\[^{10}\text{See also Duranton and Puga (2004).}\]
of labor turnover that emerge with volatile investment returns by incorporating worker heterogeneity in ability and job assignments.

3 Model

There is free entry into production, where all firms are ex ante identical and the only input is labor. A worker’s career lasts three periods. In each period the worker supplies one unit of labor inelastically. There are an infinite number of firms in the market. There are two job levels in the production hierarchy of firms – Job Level 1 and Job Level 2. Job Level 2 involves working with technically challenging R&D projects, the returns to which are stochastic.

In the first period, firms hire workers. We denote the ability of workers as \( \theta \) which varies uniformly over the interval \([0, 1]\). Firms and workers cannot observe \( \theta \). After the worker has been employed in the firm for more than one period, her ability becomes known to her current employer. Firms, however, cannot observe the ability of a new worker in any period. At the end of the first period firms invest \( I \in [0, I] \) in R&D. There is an increasing and convex cost function for R&D investment denoted by \( c(I) \) with \( c(0) = 0 \) and \( \lim_{I \to I} c'(I) = \infty \).

In every period following the investment, there is an exogenous probability, \( p \in (0, 1) \), that the firm’s technology is successful.\(^{11}\) Let \( r_{it} \) denote the returns to R&D investment. When Firm \( i \)'s technology is successful, \( r_{it} = r > 0 \), while \( r_{it} = 0 \) when it is unsuccessful. The probability, \( p \), of a successful draw in any period is independent of the outcome in the previous period.

Without loss of generality, we will refer to the current employer firm as \( i \) and a new employer as \( j \). The productivity of a worker in Job Level 1 is independent of her ability. For a worker who has been employed in Firm \( i \) for \( \tau \) periods, her output in Job Level 1 is

\[
y_{i}^{1} = 1 + s_{\tau},
\]

\(^{11}\)We implicitly assume that there are an infinite number of firms in the industry. This will ensure that there is always at least one firm that has a good return on its investment in any period. This is because the probability that all firms have zero returns on investment goes to zero. If \( n \) is the number of firms, 
\[
\lim_{n \to \infty} (1 - p)^{n} = 0
\]
where \( s_{\tau} = 0 \) for \( \tau = 0 \) and \( s_{\tau} = s \) for \( \tau > 0 \). That is, a worker who has been in a firm for more than one period accumulates firm-specific human capital of \( s \) which increases the output she produces in the current firm.

The output of worker \( k \) assigned to Job Level 2 in her current employment in Period \( t \), is

\[
y_{ijtk}^2 = \theta_k (1 + s) (r_{it} + r_{it-1}) I,
\]

where \( r_{it} \in \{0, r\} \) is the profitability of the R&D investment in period \( t \). The worker accumulates research knowledge acquired from working in R&D projects in Job Level 2 in past periods. So even if \( r_{it} = 0 \) in the current period, she can be productive in Job Level 2 applying her knowledge from her previous period experience. However a worker assigned to Job Level 2 does not produce anything if the firm gets a zero return on its investment in both Periods 2 and 3. Also note that while worker output in Job Level 1 is constant, in Job Level 2 her output depends on ability. This means that the firm will want to assign relatively higher ability workers to Job Level 2, while keeping lower ability workers in Job Level 1.\(^{12}\)

For worker \( k \) from Firm \( i \), output in a Job Level 2 position in a new firm \( j \) in Period \( t \) is

\[
y_{jikt}^2 = \theta_k (r_{jt} + \beta r_{it-1}) I,
\]

where \( \beta \) denotes the extent to which R&D knowledge acquired by the worker in Job Level 2 is substitutable across firms. We assume that \( \beta < 1 \) so that current employers with successful technology can utilize their workers’ research experience better than a new firm.\(^{13}\)

Note that there is some continuity in the utilization of past technology. The idea is that even if \( r_{i3} = 0 \) and the old technology used by the firm declines, the utilization of that technology in the firm’s production process does not disappear abruptly, rather it continues to be used in future periods. Hence the knowledge gained from working with a technology in Period 2 can be applied to production in subsequent periods even if the firm experiences

\(^{12}\)Our results do not depend on the assumption that Job Level 1 output is independent of ability and investment. We could consider a more general specification where Job Level 1 output is \([1 + \lambda \theta (r_{it} + r_{it-1}) I] (1 + s_{\tau})\), with \( \lambda < 1 \). For \( \lambda = 0 \), we are back to our original model. All our results hold as long as we assume that \( \lambda \) is small enough.

\(^{13}\)See Moen (2005) for an empirical justification of this assumption.
an adverse investment shock and the firm’s technology declines in Period 3. Similarly, if
the old technology declines, the R&D knowledge associated with that technology can be
applied towards a new technology in a new firm, but in a less than perfect way (so that
$\beta < 1$). This allows us to examine cross-period interactions between job assignments and
turnover.

We assume that the average worker is more productive in Job Level 1 in Period 2, i.e.
$\frac{1}{2} r \bar{I} < 1$. This also ensures that a worker about whom the firm has no information will
be assigned to Job Level 1. We also impose some structure on the cost function in order
to ensure that firms choose a unique and positive level of investment in the first period
equilibrium.\footnote{A sufficient condition to guarantee that the total expected surplus from a worker at the end of period 1
is concave in $I$, is to assume that costs are convex enough. In particular, we assume that $c''(I) I^3 \geq \frac{3(1+\rho)}{\rho}$
for every $I \in [0, \bar{I}]$.}

The timing of events is as follows. In the first period, firms hire workers. Worker ability
is unknown to all firms and workers although they know the distribution of abilities. Since
a worker does not produce any output in Job Level 2 in the absence of an R&D investment,
all workers are assigned to Job Level 1 in Period 1. At the end of Period 1, firms invest in
R&D. Firms’ investment levels are publicly observable. Worker ability is privately revealed
to the current employer, but is unknown to new employers in the outside market. In the
beginning of Period 2, the firms’ R&D outcome is realized and it is observed by all firms and
workers. Firms announce job assignments for each of their workers. The outside market
makes wage offers to new workers based on observed job assignments and turnover.\footnote{We do not need to make any assumptions about worker information about her own ability. Even if
the worker knows her ability as the current employer does in Period 2, there is no way for her to credibly
communicate that to a new employer. In other words every worker will want to pretend to have the highest
ability and knowing that, new employers will simply ignore any communication from the worker about her
ability. They will instead condition their beliefs on observed job assignments and turnover.} Each
firm can make counteroffers to the workers that it wishes to retain. Workers accept the
highest wage offer, and stay with the same firm if indifferent. The same sequence of events
as in Period 2, repeats in Period 3. After Period 3, the game ends.

Two aspects of our model deserve attention here. First, note that it is the transfer
of R&D knowledge in conjunction with the uncertainty in R&D outcome that induces turnover of workers in Period 3. Workers who accumulate research experience are high ability workers employed in firms with successful technology in the past. These workers become more useful in other firms with successful technology, if their old employer’s technology declines in the future. Hence we observe relatively higher ability workers moving to new firms, and the average ability of workers who change employment increases as R&D knowledge becomes more easily transferrable across firms. The R&D investment decision of the firm will then depend on the extent of turnover that occurs. Second, job assignments will be inefficient and this inefficiency arises because assigning the worker to a higher level job increases her productivity in all firms. If the firm’s technology is successful, not assigning a highly productive worker to the R&D job entails a cost in terms of lost utilization of returns from R&D investment. At the same time assigning a worker to the higher level job increases the probability that the firm will lose the worker to a new firm in the third period if it does not realize successful returns on its investment. The greater the former effect the lower will be the inefficiency in job assignments.

4 Analysis

We solve for the outcome of this model by backward induction in this section. The solution concept used is Perfect Bayesian Equilibrium.

4.1 Job Assignments and Turnover

In order to derive the job assignment and turnover outcomes for periods 2 and 3 we take as given the symmetric equilibrium investment of firms in Period 1, $I^*$. Later we derive the equilibrium investment explicitly. We represent the R&D outcome history of Firm $i$ in Period 3 by the tuple $(r_{i2}, r_{i3})$, where each takes the value of $r$ or zero depending on whether Firm $i$’s technology was successful or not in periods 2 and 3 respectively. Thus, a firm with history $(r, r)$ had successful outcomes in both periods. History $(r, 0)$ means the firm had a positive return in Period 2 but not in Period 3. The four possible histories that a firm can have in Period 3 are $(r, r)$, $(r, 0)$, $(0, r)$ and $(0, 0)$. Also, we refer to workers who
have worked in a Job Level 2 position in a firm that had successful R&D as workers with “R&D experience.”

Turnover of workers can occur only if worker productivity is higher in another firm. In firms with history \((r, r)\), and \((0, r)\), the worker is always more productive in the old employment due to better internal utilization of research experience and the firm-specific human capital accumulated by the worker. Hence turnover will never occur in these two cases. In the case where a firm has history \((0, 0)\), turnover will not occur simply because the firm does not have an incentive to assign any worker to Job Level 2 in either period and a worker with no R&D experience is always more productive with her original employer. Hence, the only case where turnover can occur is when a firm’s technology was successful in Period 2, but declined in Period 3, i.e. in firms with history \((r, 0)\). In such firms, workers may be more productive in another firm if the R&D knowledge they can transfer is high enough. Since we are interested in considering all possible ranges of \(\beta\) where promoted workers are more productive in a new firm, we assume that firm-specific human capital is low enough to allow these situations to arise. A sufficient condition to ensure this is \(s < 1\).

In the following proposition, we describe the conditions under which worker turnover can occur.

**Proposition 1** *Given the possible histories of R&D realizations, turnover occurs in Period 3 if only if a firm has R&D outcome history \((r, 0)\) and \(\beta \geq s\).*

The productivity of the worker in a new firm depends on the magnitude of \(\beta\). The greater the transferability of knowledge across firms relative to the firm-specific human capital \(s\), the greater the expected productivity of the worker in a new firm relative to her productivity in the current employment.

In order to focus on the nature of worker turnover in these firms, for the rest of the analysis in this paper, we restrict attention to the case where turnover occurs in Period 3, i.e. firms that have R&D outcome history \((r, 0)\) with \(\beta > s\). We further assume that \(\frac{1}{2} \leq p \leq 1\). This restriction ensures that the probability that a firm’s technology remains successful in the future is high enough so that a positive measure of workers is promoted in firms with successful technology in Period 2.
Proposition 2 describes the equilibrium outcome for periods 2 and 3 in a firm with R&D outcome history \((r,0)\).

**Proposition 2** For a firm with R&D history \((r,0)\), the equilibrium outcome for job assignment and turnover in periods 2 and 3 has the following properties:

(a) In Period 3, there exists \(\hat{\theta}_3 \in \left[\frac{1}{rI^*}, 1\right]\) such that a worker with ability \(\theta\) moves to a new firm if and only if \(\frac{1}{rI^*} \leq \theta \leq \hat{\theta}_3\) with a wage of \(w_3^2(r,0) = \bar{\theta}_3 (1 + s) rI^*\), where

\[
\hat{\theta}_3 = \frac{(\beta + 1)}{(1 + 2s - \beta) rI^*}.
\]

All workers with \(\theta < \frac{1}{rI^*}\) are retained in Job Level 1 at a wage of 1 and workers with \(\theta > \hat{\theta}_3\) are retained in Job Level 2 at a wage of \(w_3^2(r,0)\).

(b) In Period 2, all workers are retained in the same firm and a worker with ability \(\theta\) is assigned to Job Level 2 if and only if \(\theta \geq \frac{1}{rI^*}\). A Job Level 2 worker is paid a wage of \(w_2^2(r) = 1 + p(1 + 2s - \beta)\). Job Level 1 workers receive a wage of \(1 + s\).

Proposition 2 yields a number of insights about the characteristics of labor turnover in these firms. First, note that in Period 3 for firms with history \((r,0)\), it is not the lowest ability workers who are moving to new firms. Since a firm will assign its better workers to Job Level 2 in Period 2, such workers will gain R&D experience that can be utilized in a new firm. Hence it is these workers who switch firms \(\left(\frac{1}{rI^*} \leq \theta \leq \hat{\theta}_3\right)\).\(^{16}\) Contrary to the predictions of existing asymmetric learning models, the lowest ability workers are the ones who are the least likely to change firms. This is because in our model, turnover is efficient only if the worker has accumulated R&D knowledge from working in Job Level 2. Since lower ability workers are less likely to be assigned to R&D jobs in Period 2 they are also less likely to move in Period 3. Also, as R&D knowledge becomes more easily transferable, it becomes increasingly costly for the old firm to retain workers with R&D experience, since the wage they command in the outside market increases. This leads to greater turnover.

\(^{16}\)This result is similar to Perri (1995) where he shows that the winner’s curse phenomena that arises when a firm can make counteroffers disappears when job assignments signal ability and there is some exogenous turnover.
Second, there is no turnover in any firm in Period 2. Turnover occurs only in Period 3 after a measure of workers have acquired R&D knowledge from being employed in R&D-intensive jobs in Period 2. Thus, our results provide an explanation for the absence of a negative relationship between tenure and turnover. Since promotions occur in the later part of the worker’s career and R&D knowledge that determines her productivity in a new firm is tied to promotions, turnover does not decrease with tenure.

An intuitive result that emerges from Proposition 2 is that as long as firms retain the highest ability worker, i.e. \( \hat{\theta}_3 < 1 \), the proportion of workers who turnover decreases with the level of investment. The probability that a worker moves to a new firm is \( (\hat{\theta}_3 - \frac{1}{rI^*}) \). After substituting for \( \hat{\theta}_3 \), the expression that denotes this probability is

\[
\Pr\left(\frac{1}{rI^*} \leq \theta \leq \hat{\theta}_3\right) = \frac{2(\beta - s)}{(1 + 2s - \beta)rI^*}.
\]

The above expression is decreasing in \( I^* \). As \( I^* \) increases, the profit from retaining a worker with R&D experience in the current employment is higher. As a result, more workers are retained and turnover is lower. Thus, the model captures the negative dependence of R&D investment and turnover found by Moen (2005).

In the first period, all firms are identical with respect to the information they have concerning worker abilities. Due to competition for workers in the external labor market, firms make zero profits and all workers are paid their output in Period 1 plus the total expected profits they generate for the firm in periods 2 and 3. Since all workers are assigned to Job Level 1 by assumption, the expected output from a worker in Period 1 is 1. Suppose

---

17 The only other paper to our knowledge that models R&D investment in the presence of spillovers via worker mobility is Cooper (2001). Cooper considers a model where competing firms invest in R&D and workers can switch employers in the event of a bad match with the current employer. When the worker does so, she is able to utilize a part of the R&D investment from her old employment in the new firm. Thus worker mobility helps to internalize spillovers and this mitigates the over-investment inefficiency caused by R&D spillovers. Cooper’s model predicts that worker mobility will not depend on the level of R&D investment. By contrast, we are able to generate the dependence between turnover and investment found by Moen by incorporating the interaction between investment, job assignments, and knowledge accumulation. Firms always have the option of assigning a worker to a lower level job where there is no opportunity for R&D knowledge accumulation and possible turnover. But the loss from misassigning a worker to prevent turnover is directly influenced by the level of investment in the firm.
the expected profit from a worker in periods 2 and 3 is given by \( \pi(I^*) \), then the first period wage is \( w_1 = 1 + \pi(I^*) \).

## 4.2 Investment in R&D

In this subsection we derive the equilibrium R&D investment in Period 1. Given the structure we have imposed on costs, the R&D investment chosen by firms will be high enough to make it worthwhile to assign a positive measure of workers to Job Level 2 if the firm has a successful technology in any period. Let \( \pi^{r_2 r_3} \) denote the expected profit across periods 2 and 3 from a worker if the realized R&D outcome is \( (r_2, r_3) \). Then, the total expected profit from investing \( I \) in R&D in Period 1 is

\[
\pi(I) = (1 - p)^2 \pi^{00}(I) + p (1 - p) \left[ \pi^{0r}(I) + \pi^{r0}(I) \right] + p^2 \pi^{rr}(I) - c(I).
\]

In the following proposition we compare the equilibrium level of investment and promotion with the efficient level. We assume that the profit function is such that a positive measure of workers are promoted in Period 2 at the equilibrium level of investment chosen by firms in Period 1.\(^{18}\)

**Proposition 3** The equilibrium levels of investment and job assignment are inefficient and the inefficiency increases as \( \beta \) increases.

Period 2 job assignments are inefficient in firms that had successful technology that period because assigning a worker to the R&D job raises the wage that she can command in the future due to the R&D knowledge she acquires that can be potentially utilized in a new firm. Hence, the firm’s incentive to assign workers to R&D jobs is lower than the efficient incentive.

Note that the transferability of R&D knowledge represented by \( \beta \) has a salutary effect on the efficient investment level in the industry since it increases the productivity of R&D workers in Period 3. On the other hand, higher levels of \( \beta \) lowers the investment level chosen by the firms. Hence, as \( \beta \) increases, the difference between the efficient investment level and the equilibrium level widens. Now as investment levels fall due to higher \( \beta \), fewer

\(^{18}\) See Proof of Proposition 3 in the Appendix for a detailed description of the conditions.
workers are assigned to R&D jobs. On the other hand, efficiency requires that more workers should be assigned to Job Level 2, both due to a direct effect on output of an increase in $\beta$ and also due to the consequent increase in the efficient investment level in Period 1. This means that the efficient and equilibrium promotion cutoffs also diverge as $\beta$ increases.

The effect of knowledge transfer on the promotion cut-off provides an interesting result concerning the relationship between $\beta$ and the average ability of the turnover pool in Period 3. This is formalized in the following proposition.

**Proposition 4** The average ability of a worker from a firm with R&D outcome history $(r, 0)$ who changes employers in Period 3 increases as $\beta$ increases.

Equilibrium investment decreases as $\beta$ increases. This is intuitive since $\beta$ only affects the wages that have to be paid out in Period 3. The adverse effect on equilibrium investment means a lower profit from assigning a worker to Job Level 2 in Period 2 and hence $\hat{\theta}_2$ increases. At the same time, as $\beta$ increases the cut-off ability level of the worker who changes employment increases, i.e. $\hat{\theta}_3$ increases. Thus, as R&D knowledge becomes more transferable across firms, we are likely to find better and better workers changing employers.

### 4.3 Turnover Efficiency and Complete Turnover of R&D Workers

Asymmetric information about worker abilities in the presence of knowledge transfer also leads to a third kind of inefficiency, namely inefficient turnover resulting from a misallocation of workers across firms. Specifically, consider the outcome in firms with R&D history $(r, 0)$ in Period 3. All workers with R&D knowledge from such firms are more productive in a firm that has successful R&D returns in Period 3. Hence efficient turnover will mean that all R&D workers in these firms with $r_{i3} = 0$ should turnover. In the following analysis, we look at conditions that lead to turnover of all promoted workers in Period 3. This will yield a second-best outcome for turnover in Period 3 given the level of investment and promotions that occurred in the past. 19

The nature of the relationship between knowledge transfer, turnover and investment highlighted in propositions 3 and 4 suggests that for $\beta$ high enough, firms that realize

---

19 Allocation of workers across firms is inefficient in Period 2 as well. For example, high ability workers in firms with $r_{i2} = 0$, should be assigned to Job Level 2 positions in firms that have $r_{i2} = r$. 15
R&D outcome \((r, 0)\) in Period 3 may not find it worthwhile to retain any promoted worker. In this case, all workers with \(\theta \geq \frac{1}{\gamma r}\) will switch employment to a new firm leading to complete turnover of all R&D workers. In other words, given the set of workers who are promoted, the allocation of workers across firms in Period 3 becomes efficient.\(^{20}\)

Under our assumptions, the highest level of \(\beta\) is 1. This will be the case when R&D knowledge is perfectly substitutable across different firms. Let \(\widehat{I}\) represent the lowest level of investment such that a firm with history \((r, 0)\) finds it worthwhile to retain a positive measure of workers with R&D experience when \(\beta = 1\). Then such a firm will not find it worthwhile to retain any worker with R&D experience in Period 3 if the equilibrium investment level falls below \(\widehat{I}\). The equilibrium level of investment will be lower than \(\widehat{I}\) if the marginal cost of investment is high enough. If we assume that this is not the case, then the composition of turnover will depend both on \(\beta\) and on the probability of a positive return on R&D investment. The following proposition formalizes the conditions under which we will see complete turnover of R&D workers.

**Proposition 5** In Period 3, we will see complete turnover of all high ability R&D workers in the worker turnover pool if the following conditions hold.

(a) \(\beta\) is high relative to \(s\), and

(b) \(p\) is neither too high nor too low.

The above proposition suggests that we should observe turnover of high ability workers in industries where knowledge is easily transferable across firms and the variance in returns to investment is high. For \(\beta\) high enough, the wage that needs to be paid to retain workers with R&D experience in firms that have outcome \((r, 0)\) is greater than their productivity in the firm. Thus, all high ability workers with R&D experience turnover. The intuition for why variability in returns to investment is important for complete turnover of R&D workers is as follows. Consider a situation where \(p\) is too high, so that the equilibrium level of investment in Period 1 is very high. When this is the case, the firm with R&D history \((r, 0)\) in Period 3 will want to retain some of its workers with R&D experience.

\(^{20}\)Investment and job assignment inefficiency are aggravated since the loss from turnover reduces incentives for investing in and utilizing R&D.
On the other hand if \( p \) is too low and hence investment is very low, firms with successful returns in Period 2 will not find it worthwhile to assign any worker to the R&D-intensive job. In this case no worker will have R&D experience which means no turnover will occur in Period 3. Thus, complete turnover of R&D workers is observed when \( p \) is high enough so that it is profitable to assign workers to the R&D job in Period 2, but \( p \) is not high enough to induce large investments that make it worthwhile to retain R&D workers in Period 3 if R&D returns are zero. Intermediate values of \( p \) in turn correspond to high variance in R&D returns. Since high-technology industries frequently consist of numerous small firms, especially start-ups, with greater unpredictability in returns from innovation, the predictions from Proposition 5 provide an explanation for the observed nature of labor turnover across these firms.

There has been much discussion in the literature on job-hopping in technology industries about how easy mobility has facilitated the success of this industry. In particular many researchers have discussed the role of labor mobility in drawing a contrast between the decline of Route 128 technology firms in Massachusetts and the roaring success of the same industry in Silicon Valley. Saxenian (1994) identified cultural differences across the two clusters as the primary reason for labor mobility while Gilson (1999) and Hyde (2003) point to differences in the legal infrastructure and in particular the unenforceability of non-compete clauses in California. At the same time, there is also a vast literature that looks at the formation of industrial clusters especially in high-technology industries. These papers identify the importance of labor pooling as a factor driving competing firms to locate close to each other.\(^{21}\)

While examining the reasons that lead to the easy movement of labor in Silicon Valley and other high-technology clusters is beyond the scope of this paper, we can use our framework to describe when and why labor mobility may facilitate industry growth. Our model predicts that this is more likely to be the case when we have complete turnover of high ability R&D workers under the conditions described in Proposition 5. Let us denote the cut-off level, \( \beta \in [s, 1] \), as the level of \( \beta \) identified by Proposition 5 above which there is complete turnover of R&D workers in Period 3. Let \( \delta_1 \) and \( \delta_2 \), \( \delta_1 < \delta_2 \), denote the level of

\(^{21}\)Gerlach et. al. (2009); Combes and Duranton (2006); Fosfuri and Ronde (2004).
restrictions to labor mobility in two otherwise identical industries (or technology clusters). We can think of Industry 1 as the Silicon Valley cluster and Industry 2 as the Route 128 firms. Then the effective level of R&D knowledge transfer possible across firms in Industry $k \in \{1,2\}$ is given by $\beta - \delta_k$. Let $Y_k^*$ denote the total expected equilibrium output in Industry $k$ across all periods.

**Proposition 6**

a) When there is complete turnover of all R&D workers in the turnover pool in Period 3, i.e. $\beta - \delta_k \geq \bar{\beta}$ for every $k$, greater restrictions on labor mobility will lower industry output so that $Y_1^* \geq Y_2^*$.

b) In the absence of complete turnover in Period 3, i.e. if $\beta - \delta_k < \bar{\beta}$, for any $k$, the effect of labor mobility restrictions is ambiguous and $Y_1^* \leq Y_2^*$.

Before we explain part a) of the above proposition it is useful to understand the output effect of $\delta_k$ when $\hat{\theta}_3 < 1$ and the $(r,0)$ firms retain some of their R&D workers in Period 3. In this case, labor market restrictions will affect the firm’s investment choice in Period 1. This is because with a higher $\delta_k$ and consequently a lower $\beta - \delta_k$, the firm will expect lower turnover and hence higher profits from its R&D workers in Period 3. Since the firm can thus appropriate more of its investment from R&D workers, $\frac{dI^*}{d\delta_k} > 0$. This, investment effect will tend to increase industry output. On the other hand, lower turnover also means greater turnover inefficiency which dampens industry output. Without making additional restrictions on the parameters and the investment cost function, it is not possible to say which of these effects is stronger.

However, when all R&D workers move to a new firm, the investment effect of labor mobility restrictions disappears. This is because, with all Job Level 2 workers leaving the firm in Period 3 with an $(r,0)$ outcome history, the firm does not expect any profits from its R&D workers in that event. Then the only effect of labor mobility restrictions is that it reduces the utilization of R&D knowledge by a worker in a new firm and hence necessarily lowers industry output.

Proposition 6 provides an explanation for why the high-technology industry in Silicon Valley appears to have outperformed that in Route 128. The extensive use of non-compete contracts facilitated by permissive Massachusetts law restricted labor mobility between
Route 128 firms and hindered the efficient allocation of highly productive technology workers across firms in that cluster.

At the same time our results also provides caution against blanket recommendations in favor of labor mobility. In particular, we believe that industry dynamics in terms of investment volatility and knowledge transmission across firms play an important role in determining the appropriate level of labor mobility for an industry. To our knowledge most of the studies that have examined the effect of legal restrictions on labor mobility have focused only on high-technology industries. (Samila and Sorenson, 2011; Marx et. al. 2009). As our results suggest, high-technology industries are likely to experience, both very high rates of turnover among productive workers as well as positive industry outcomes resulting from such turnover. Both of these features are driven by underlying industry characteristics namely, the volatile nature of investment in technology and the transfer of R&D knowledge across firms through worker mobility. By restricting the analysis to high-technology industries, existing studies do not capture the role of these crucial industry-specific factors in explaining the relationship between labor mobility and industry growth. Thus our analysis calls for cross-industry comparisons on the effects of labor mobility restrictions accounting for specific industry characteristics that drive labor turnover there.

5 Discussion and Conclusion

There has been substantial research on the high rates of labor mobility observed in R&D intensive innovative industries such as the high-technology firms of Silicon Valley and other technology clusters. An important feature highlighted by these studies is the transmission of knowledge facilitated by such mobility across firms in these industries. Many papers have established the fact that engineers and scientific personnel do transfer crucial knowledge across firms.22 At the same time, there is also considerable evidence that these high-technology firms are subject to persistent technological shocks.23 This creates a continuous cross-section of expanding and contracting firms making labor turnover efficient. We argue in this paper that the combination R&D knowledge transferred across firms through labor

---

22Moen (2005); Almeida and Kogut (1996).
mobility and the volatile nature of high-technology markets together create conditions for the unique patterns of labor turnover observed in high-technology firms and also yield distinctive consequences of such turnover for overall productivity and growth of the industry. In the current section we discuss our results more broadly and draw a contrast between the experience of the high-technology industry with other knowledge-creating industries as well as with other industries where firms experience worker turnover as a result of stochastic productivity shocks. We also describe how our theory and predictions differ from other theories of turnover.

There is a wide range of professional service industries that one could classify as knowledge-intensive, such as law and accounting firms, financial service firms and consulting firms. As with high-technology firms, employees in such occupations embody a great deal of knowledge acquired through their work experience in the current employment. However, these markets generally face relatively stable market demand as well as more or less standardized production processes. While these firms may be subject to industry-wide macroeconomic shocks that displace workers from time to time, they do not experience continuous and firm-specific investment shocks in a manner that high-technology firms do. As a result most of these firms appear to have strong internal labor markets or up-or-out promotion ladders with low overall turnover rates.24

For example, studies of financial firms, most notably the Baker Gibbs Holmstrom’s (BGH, 2001) single firm study, establish the presence of strong internal labor markets in such firms. (See also Eriksson and Werwatz, 2005). To be sure, the BGH study does find external hiring at all job levels within the firm. However, they also confirm the existence of long careers within the firm with relatively stable employment for a substantial majority of the employees.

While it is difficult for us to assess the exact significance of their finding as it relates to our model, we note the following points. First, since the BGH study only looks at one single firm, they are not able to track the employment experience of workers who enter or leave the firm at various job levels. To the extent that entering and exiting workers took a significant wage cut in their new employment or moved to unemployment after the separation, the experience of this firm will be explained more accurately by traditional models of asymmetric employer learning and exogenous turnover. In fact the authors also believe this to be the case. Second, we do not necessarily suggest that high-technology firms are the only ones to experience a decline in internal labor markets. There is a general view that many firms in several industries are moving to a greater dependence on external markets and spot wage determination. At the same time internal labor markets appear to have shown a greater decline in high-technology industries than

24
Several industries, apart from high-technology markets, experience volatile demand or supply shocks that cause some firms within the industry to expand and others to contract. Andersen and Meyer (1994) show that there is significant job turnover caused by simultaneously expanding and contracting firms. For example they find that the manufacturing sector has relatively high worker turnover, but most of the turnover is characterized by temporary separations where the worker returns to the employer after a period of time. This points to the importance of firm-specific human capital in such industries and a limited role for exporting the worker’s human capital to new firms. Further in both manufacturing and other sectors, such as retail trade and services, that had high separation rates, there were significant earnings losses for workers and rehiring costs for firms as a result of turnover. Thus, quite unlike high-technology markets, it appears as if turnover imposes significant costs on firms and workers in other industries where knowledge transmission through worker mobility is mostly absent.

Finally we note that even within the high-technology industry, there is some variation in the volatility of investment and the extent of labor mobility. While we have referred to high-technology industry and R&D intensive firms as a homogenous block, it is important to look at the sub-sectors at a disaggregated level in order to correctly apply our results. For example, Fallick et. al. note that while the computer industry clusters in California exhibited significantly higher rates of job-hopping than similar clusters in other states, these differences were not significant outside the computer industry. As these authors point out, this is likely the result of the modular production style adopted by the computer industry that exposes these firms to greater technology shocks than other industries.\(^{25}\) This again confirms our hypothesis about the nature of labor turnover being driven by volatility in investment returns.

Our analysis provides some useful directions for future empirical study of labor turnover. In spite of an abundance of case studies and interview-based studies of high-technology in other sectors. (DiPrete et. al. 2002; Osterman and Burton, 2004).

\(^{25}\)Modular production means that computer manufacturers rely on independent suppliers for individual components. Since these suppliers pursue their own innovation strategies, the rate of technical advancement in the computer industry is quite high.
firms in the organizational science and sociology literature, there have been few systematic empirical studies of the patterns of turnover or its effects on industry growth in the R&D-intensive technology sector. Fallick et. al. provide a good analysis of the reasons driving high labor mobility in Silicon Valley. The results of their study conform to a number of assertions brought out by our model. At the same time their results suggest that there are unique aspects of the computer industry that interact with the legal environment in California to make job-hopping so common. In light of their findings and our own, we believe that, in order to better understand the extent and nature of turnover in high-technology firms, it is important to examine in greater detail, the effect of industry-level characteristics, such as R&D investment, technological volatility, and the importance of general and specific human capital accumulation. As we show in this paper, the causes and effects of turnover are tightly linked. Hence policies that facilitate or restrict the movement of labor across firms are likely to have different implications for industry growth depending on the nature of labor turnover in that particular industry. This calls for further empirical study of the effects of labor mobility on industry growth controlling for the underlying industry-level characteristics that influence the patterns of such labor turnover.

6 Appendix

Proofs of Propositions 1 & 2. We present the proofs of Proposition 1 & 2 together since they are related. We first derive the equilibrium in Period 3 and then work backwards. Before that we solve the cutoff $\beta$ as given in Proposition 1. In firms with R&D outcome history $(r, r)$, all workers are more productive in their current employment in both periods. Hence, there is no worker turnover from such firms in equilibrium. In firms with R&D history $r_{2} = 0$ no turnover occurs since all workers are assigned to Job Level 1 in Period 2, and no new firm will be willing to assign a completely unknown worker to Job Level 2 given our assumption that $\frac{1}{2}rT < 1$. Hence, turnover can occur only if a firm has outcome history $(r, 0)$ and workers who accumulated research experience from holding Job Level 2 positions in that firm are more productive in a new firm that has a good R&D draw in Period 3. This will happen when $\theta (1 + s) rI^* \leq \theta (1 + \beta) rI^*$, which means that $\beta \geq s$. 

22
When $\beta \geq s$, given that a positive measure of workers was promoted in Period 2, current employers with history $(r, 0)$ will not find it profitable to retain the lowest ability worker who was promoted since she will have to be paid a wage that is higher than her productivity. Hence, there will be a set of relatively lower ability workers among the promoted workers who will turnover.

2 (a) Now suppose workers with $\theta \geq \hat{\theta}_2$ were assigned to Job Level 2 in Period 2. These workers are more productive in a new firm in the current period since $\beta \geq s$. But since the original firm has an informational advantage about the ability levels of its own workers, it will try to retain the best of its promoted workers. Suppose the market believes that a promoted worker who comes to the firm for employment has expected ability $\theta^e$, the outside wage for the promoted worker will be $\max \{\theta^e (1 + \beta) r I^*, 1\}$. As long as $\hat{\theta}_2 \geq \frac{1}{(\beta + 1) r I^*}$, the outside wage for a promoted worker will be $\theta^e (1 + \beta) r I^*$. In the next part of the proposition we show that this true. At this outside wage, Firm $i$ will retain a worker of ability $\theta$ if and only if $\theta (1 + s) r I^* \geq \theta^e (1 + \beta) r I^*$.

The cutoff level of ability above which a worker is retained is $\hat{\theta}_3 = \frac{\theta^e (1 + \beta)}{1 + s}$. Then, since a worker is retained if and only if $\theta \geq \hat{\theta}_3$, the conditional expected ability of the worker is $\theta^e = \frac{\theta_2 + \theta_3}{2}$. Solving for $\hat{\theta}_2$ we get

$$\hat{\theta}_3 = \hat{\theta}_2 \frac{(\beta + 1)}{1 + 2s - \beta} > \hat{\theta}_2.$$

Thus, workers with $\theta \in [\hat{\theta}_2, \hat{\theta}_3]$ move to a new firm. The wage for workers with $\theta \geq \hat{\theta}_2$ is

$$\frac{1 + s}{1 + 2s - \beta} (1 + \beta) \hat{\theta}_2 r I^*$$

The Period 2 promotion cut-off, $\hat{\theta}_2$ is derived in the next part of the proof as $\frac{1}{r I^*} \geq \frac{1}{(\beta + 1) r I^*}$. By substituting the value of $\hat{\theta}_2$ we get the wage as given in the proposition. Workers who were assigned to Job Level 1 in Period 2, i.e. those with $\theta < \hat{\theta}_2$ will receive an outside wage according to their productivity in Job Level 1 in a new firm, which is 1.

(b) In Period 2 a worker employed in a firm with $r_{i2} = r$ is always more productive in Firm $i$ than in a new firm. Hence all workers are retained in the same firm and there will be no turnover.
Given that \( p \geq \frac{1}{2} \), for every \( \beta < 1, \beta < \frac{p(1+s)-(1-p)s}{p} \). The wage for a promoted worker in the outside market is determined by her expected productivity in a Job Level 2 position in a new firm. So if the market believes that workers with \( \theta \geq \tilde{\theta}_2 \) are promoted in firms with \( r_{i2} = r \), then the wage for a promoted worker will be \( \tilde{\theta}_2 r I^* + p\tilde{\theta}_2 r I^* (1 + 2s - \beta) \) which is the total expected output of the least productive worker in a new firm. This is because, in Period 2 the expected output of all workers is higher in the current firm than in a new firm and the current employer can make counter wage-offers. Under these conditions, beliefs about worker ability in a new firm will be driven by a winner’s curse result. To see why consider what happens if the new firm offers a wage that is higher than the expected output of \( \tilde{\theta}_2 \). The current employer will make a counteroffer to retain a worker at this wage if and only if the expected output of the worker in the current firm is higher than this market wage. This in turn means that workers who move to a new firm had an expected output lower than this market wage, so that the new firm will necessarily make a loss on the new workers who turnover at this wage. Thus the only belief about worker ability that is consistent in equilibrium is that it is \( \tilde{\theta}_2 \).26

A worker who is not promoted will be paid a wage of \( 1 + s \), which is the total expected output of a worker in Job Level 1 across periods 2 and 3. Comparing the profit from promoting a worker with that from keeping her in Job Level 1, we obtain the cut-off, \( \tilde{\theta}_2 = \frac{1}{r I^*} \). Substituting for \( \tilde{\theta}_2 \) in the wage expression above, we get \( 1 + p(1 + 2s - \beta) \) as the Job Level 2 wage. ■

**Proof of Proposition 3.** First we derive the equilibrium investment in Period 1. The expected profit from a worker in the following two periods for each possible R&D outcome

---

26 See Golan (2005) for a detailed explanation of this result.
vector is given below.

\[
\begin{align*}
\pi^{00} (I) &= s \\
\pi^{0r} (I) &= \int_0^{\tilde{\theta}_2} s d\theta + \int_{\tilde{\theta}_2}^{1} \left[ \theta (1 + s) r I - 1 \right] d\theta \\
\pi^{r0} (I) &= \int_0^{\tilde{\theta}_2} s d\theta + \int_{\tilde{\theta}_2}^{1} \left[ \theta (1 + s) r I - w^2_I (r) \right] d\theta + \int_{\tilde{\theta}_3}^{1} \left[ \theta (1 + s) r I - w^2_I (r, 0) \right] d\theta \\
\pi^{rr} (I) &= \int_0^{\tilde{\theta}_2} s d\theta + \int_{\tilde{\theta}_2}^{1} \left[ 3\theta (1 + s) r I - w^2_I (r) - (\beta + 1) \right] d\theta
\end{align*}
\]

Then, the total expected profit from investing \( I \) in R&D in Period 1 is

\[
\pi (I) = (1 - p)^2 \pi^{00} (I) + p (1 - p) \left[ \pi^{0r} (I) + \pi^{r0} (I) \right] + p^2 \pi^{rr} (I) - c (I).
\]

Under our assumption on the convexity of the cost function, the above profit function will be concave. We also need to ensure that the equilibrium investment level \( I^* \) is positive. If \( I < \frac{1}{r} \), even the highest ability worker is more productive in Job Level 1 so that it is not worthwhile to promote any worker in any period. In this case the profit in every contingency following the investment is \( s - c (I) \) which is strictly decreasing in \( I \) and \( I^* = 0 \).

In order to ensure that \( I^* > 0 \) we assume that there exists an investment level, \( \tilde{I} > \frac{1}{r} \) such that \( \pi (\tilde{I}) = s \). Then the equilibrium level of investment \( I^* \) solves \( \pi' (I^*) = 0 \).

Next we derive the optimal investment, which is obtained by maximizing the total expected surplus from efficient assignment and turnover following the investment. This represents the first-best outcome in the market. The output from investing \( I \), for each
R&D outcome vector is given below.

\[ Y^{00} (I) = Y^{0r} (I) = \int_0^1 \frac{\partial_2^o (0)}{\partial_2 (0)} 2 (1 + s) d\theta + \frac{1}{\partial_2 (0)} \int_0^1 \theta r I [1 + 2 p (1 + s) + (1 - p) (1 + \beta)] d\theta, \]

\[ Y^{r0} (I) = \int_0^1 \frac{\partial_2^o (r)}{\partial_2 (r)} 2 (1 + s) d\theta + \frac{1}{\partial_2 (r)} \int_0^1 \theta r I [2 + s + \beta] d\theta, \]

\[ Y^{rr} (I) = \int_0^1 \frac{\partial_2^o (r)}{\partial_2 (r)} 2 (1 + s) d\theta + \frac{1}{\partial_2 (r)} \int_0^1 3 \theta r I (1 + s) d\theta. \]

where,

\[ \partial_2^o (0) = \frac{2 (1 + s)}{[1 + 2 p (1 + s) + (1 - p) (1 + \beta)] r I}, \]

\[ \partial_2^o (r) = \frac{2 (1 + s)}{[(1 + s) (1 + 2 p) + (1 - p) (1 + \beta)] r I}. \]

These are the efficient Period 2 promotion cutoffs for each realization of \( r_{i2} \). Note that since wages are transfers between firms and workers and there are no other production costs in the model, output is equal to surplus in periods 2 and 3. So we obtain the efficient Period 2 promotion cutoffs by comparing the total output from assigning a worker to Job Level 1 in the same firm to the total output from assigning her to Job Level 2 in the same firm if \( r_{i2} = r \) or Job Level 2 in a new firm with a successful Period 2 outcome if \( r_{i2} = 0 \). If \( r_{i2} = 0 \), then workers with \( \theta \geq \partial_2^o (0) \) should move to a firm with successful R&D in Period 2, while the remaining workers should stay in the same firm. If the firm has a successful draw in Period 3, all workers should be retained, and workers should be assigned to Job Level 2 if and only if \( \theta \geq \partial_2^o (r) \). The total expected surplus from investing \( I \) is

\[ Y (I) = (1 - p)^2 Y^{00} (I) + p (1 - p) [Y^{0r} (I) + Y^{r0} (I)] + p^2 Y^{rr} (I) - c (I). \]

Given our restriction on the convexity of the cost function, \( Y (.) \) is concave in \( I \). Let \( I^o \) denote the optimal investment. Then \( I^o \) solves \( Y' (I^o) = 0 \) We assume that the \( I^o \) lies in the range where \( \partial_2 (r) < \partial_2^o (0) < 1 \) so that a positive measure of workers are assigned to Job Level 2 in Period 2 at the efficient outcome. Some extensive algebra shows that
\(Y'(I^*) > 0\). This implies that \(I^0 > I^*\). \(^27\) Also \(\frac{\partial Y'(I)}{\partial \beta} > 0\) so that \(\frac{\partial I^0}{\partial \beta} > 0\) while \(\frac{\partial I^0(I)}{\partial \beta} < 0\) so that \(\frac{\partial I^0}{\partial \beta} < 0\). This means that as \(\beta\) increases, the equilibrium level of investment diverges more and more from the optimal level, i.e. as \(\beta\) increases \(I^0 - I^*\) increases.

Next we show that the equilibrium job assignment is inefficient. First note that for any given level of investment a positive measure of workers ought to be assigned to Job Level 2 in a new firm in Period 2 when \(r_{i2} = 0\). However, in equilibrium all workers are retained in Job Level 1 in the same firm. If \(r_{i2} = r\), \(\tilde{\theta}_2 (r) < \frac{1}{1/27} = \tilde{\theta}_2\), hence fewer workers are assigned to Job Level 2 than is efficient for maximizing output at any given level of investment. Further, since \(I^* > I^0\) and \(\frac{\partial \tilde{\theta}_2^o (r)}{\partial \beta} < 0\), \(\tilde{\theta}_2^o (r, I^0) < \tilde{\theta}_2^o (r, I^*) < \frac{1}{1/27}\).

In order to see how \(\beta\) affects the promotion inefficiency, note that since \(\frac{\partial I^0}{\partial \beta} < 0\), \(\frac{\partial}{\partial \beta} \left( \frac{1}{1/27} \right) > 0\). On the other hand \(\frac{\partial \tilde{\theta}_2^o (r)}{\partial \beta} < 0\) both because of the direct effect of \(\beta\) on \(\tilde{\theta}_2^o\) at any given level of investment as well the positive effect on optimal investment. Thus as \(\beta\) increases the difference between the equilibrium and optimal promotion cutoffs for firms with \(r_{i2} = r\) increases. For firms with \(r_{i2} = 0\), \(\frac{\partial \tilde{\theta}_2^o (0)}{\partial \beta} < 0\), so again the promotion inefficiency increases with \(\beta\). \(\blacksquare\)

**Proof of Proposition 4.** The average ability of a worker who changes firms in Period 3 from a \((r, 0)\) firm is \(\tilde{\theta}_2 + \frac{\tilde{\theta}_3}{2} = \frac{(1+s)\tilde{\theta}_2}{1+2s-\beta}\). Since \(\frac{\partial \tilde{\theta}_2}{\partial \beta} > 0\), it follows that \(\frac{\partial}{\partial \beta} \left( \frac{\tilde{\theta}_2 + \tilde{\theta}_3}{2} \right) > 0\). \(\blacksquare\)

**Proof of Proposition 5.** \(\tilde{\theta}_3\) is increasing in \(\beta\) and at \(\beta = 1\) there is complete turnover of all promoted workers if \(\tilde{\theta}_3 \geq 1\). This will be true for \(I^* \leq \hat{I}\), where

\[
\hat{I} = \frac{1}{sr}.
\]

We can show that \(\pi' (\hat{I})\) is increasing in \(p\).

\[
\pi' (\hat{I})_{p=\frac{1}{2}} = \frac{5}{8} r (1 + s)^2 (1 - s) - c' (\hat{I})
\]

\[
\pi' (\hat{I})_{p=1} = \frac{3}{2} r (1 + s)^2 (1 - s) - c' (\hat{I})
\]

Let us define \(\psi_1\) and \(\psi_2\) as

\[
\psi_1 = \frac{5}{8} r (1 + s)^2 (1 - s)
\]

\[
\psi_2 = \frac{3}{2} r (1 + s)^2 (1 - s)
\]

\(^27\) A more detailed proof of Proposition 2 is available from the author upon request.
where $\psi_1 < \psi_2$. Then if $c'(\bar{I}) > \psi_2$, then for all $p \in \left[\frac{1}{2}, 1\right]$, $\pi'(\bar{I}) < 0$ so that $I^* < \bar{I}$, and $\hat{\theta}_3 \geq 1$ for $\beta$ high enough. If $\psi_1 \leq c'(\bar{I}) \leq \psi_2$, then define $\bar{p}$ as the $p$ that solves $\pi'\left(\bar{I}; \bar{p}\right) = 0$. In this case, $\pi'(\bar{I}) \leq 0$ if and only if $p \leq \bar{p}$. Thus for $\frac{1}{2} \leq p \leq \bar{p}$, we again have that $I^* \leq \bar{I}$, so that $\hat{\theta}_3 \geq 1$ for $\beta$ high enough and all promoted workers turnover when $(r, 0)$ occurs. ■

**Proof of Proposition 6.** Let us define $\beta_k = \beta - \delta_k$.

a) From Proposition 5, let us consider the case where all promoted workers move to a new firm if their current employer has $(r, 0)$ outcome. First we show that the equilibrium investment in Period 1 does not depend on $\beta_k$. Here the equilibrium investment solves, $\pi'(I^*) = 0$, or

$$p^2 \int_{\bar{\theta}_2}^1 3 \theta (1 + s) \, r \, d\theta + 2p(1-p) \int_{\bar{\theta}_2}^1 \theta (1 + s) \, r \, d\theta - c'(I^*) = 0.$$

Note that $I^*$ is independent of $\beta_k$. Hence job assignments and $\bar{\theta}_2$ are also independent of $\beta_k$.

Now let us look at the effect of $\beta_k$ on post-investment industry output from $I^*$.

$$\frac{dY_k(I^*)}{d\beta_k} = p(1-p) \left[ \int_{\bar{\theta}_2}^1 \theta r I^* \, d\theta \right] > 0.$$  

Since the direct effect of $\beta_k$ on output is positive, and the indirect effect through investment and job assignment is absent, output is higher if $\beta_k$ is lower. ■

b) Now let us see what happens when some promoted workers are retained in the old firm. Then $I^*$ solves $\pi'(I^*) = 0$ or

$$p^2 \int_{\bar{\theta}_2}^1 3 \theta (1 + s) \, r \, d\theta + p(1-p) \left[ 2 \int_{\bar{\theta}_2}^1 \theta (1 + s) \, r \, d\theta - \frac{d\hat{\theta}_3}{dI} \int_{\bar{\theta}_3}^1 (1 + s) \, r I^* \, d\theta \right] - c'(I^*) = 0.$$

In this case, $\beta_k$ affects the profits that the firm makes on its retained Job Level 2 workers, and hence it affects the equilibrium investment. It can be checked that $\frac{dI^*}{d\beta_k} \pi'(I) < 0$. Hence $\frac{dI^*}{d\beta_k} < 0$. Differentiating $\pi'(I^*)$ with respect to $\beta_k$. As $I^*$ industry output decreases. Moreover promotion efficiency also worsens and hence industry output further decreases. At the same time, the direct effect of $\beta_k$ on output is still positive. Hence, the overall effect is ambiguous.
References


