STANDARD PROMOTION PRACTICES

VERSUS UP-OR-OUT CONTRACTS

by

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ABSTRACT

In most firms a worker in any period is either promoted, left in the same job, or fired (demotions are typically rare), and there is no specific date by which a promotion needs to occur. In other employment situations, however, up-or-out contracts are common, i.e., if a worker is not promoted by a certain date the worker must leave the firm. This paper develops a theory that explains why and when each of these practices is employed. Our theory is based on asymmetric learning in labor markets and incentives associated with the prospect of future promotion. Our main result is that firms employ up-or-out contracts when firm-specific human capital is low while they employ standard promotion practices when it is high. We also find that, if firms can commit to a wage floor for promoted workers and effort provision is important, then up-or-out contracts are employed when low-level and high-level jobs are similar. We believe these results are of interest because they are consistent with many of the settings in which up-or-out is typically observed such as law firms and academic institutions.
I. INTRODUCTION

This paper is concerned with heterogeneity in promotion practices across firms. Most firms are characterized by what we will refer to as standard promotion practices, i.e., in any period a worker is either promoted, left in the same job, or fired (demotions are typically rare), and there is no specific date by which a promotion needs to occur. In other employment situations, however, up-or-out contracts are common, i.e., if a worker is not promoted by a certain date the worker must leave the firm. There is an extensive theoretical literature on each type of promotion practice, but only a few papers have considered why standard promotion practices are employed in some settings while up-or-out contracts are employed in others.\(^1\) This paper develops a theoretical framework that allows us to investigate when each of these practices is employed.

Our analysis employs the ideas of asymmetric learning in labor markets and incentives associated with the prospect of future promotion and, in particular, builds on the earlier analyses of Lazear and Rosen (1981), Waldman (1984), and Zabojnik and Bernhardt (2001). Asymmetric learning in labor markets, which was introduced to the literature in Waldman’s paper and Greenwald (1986), is the idea that a worker’s current employer acquires more accurate information about the worker’s ability than do other potential employers.\(^2\) An important idea in this literature is the promotion-as-signal hypothesis that first appeared in Waldman’s paper. That is, because other employers directly receive less accurate information about a worker’s ability than the worker’s current employer, these other firms use a worker’s job assignment as a signal of his or her ability. This means that when a promotion occurs, other employers infer the worker is of high ability and thus increase the amount they are willing to pay the worker. There are two resulting implications. First, consistent with empirical evidence, in order to stop the worker from being bid away, a firm significantly increases a worker’s wage when a promotion takes

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1 For surveys that discuss these literatures see Sattinger (1993), Gibbons and Waldman (1999a), and Prendergast (1999).

2 Other papers that investigate asymmetric learning in labor markets from a theoretical standpoint include Milgrom and Oster (1987), Bernhardt and Scoones (1993), Bernhardt (1995), Chang and Wang (1996), Owan (2004), and Golan (2005). On the empirical side, Gibbons and Katz (1991) were the first to empirically test for asymmetric learning in labor markets and they find evidence that supports the approach. A number of more recent papers such as Pinkston (2004), DeVaro and Waldman (2004), and Schonberg (2004) have also looked at this issue empirically and find supporting evidence.
place. Second, in order to avoid sending a positive signal and thus having to pay a higher wage, a firm promotes less workers than is first-best efficient.

The incentives associated with the prospect of future promotion were first formally explored in the literature on labor-market tournaments, where the seminal work in that literature is Lazear and Rosen (1981). In that paper workers compete for a promotion through their effort choices, and the firm commits to wages for both the high- and low-level jobs before effort levels are chosen. More recently, Zabojnik and Bernhardt (2001) extend Waldman’s analysis to show that the promotion-as-signal hypothesis can be used to model incentives associated with promotion without assuming that firms can commit to future wage levels (see also Gibbs (1995) for a related analysis). They introduce a human-capital-investment choice for young workers in a model in which promotions serve as a signal. Their basic finding is that, because of the signal associated with being promoted and the subsequent high wage due to the signal, a young worker has an incentive to invest in human capital in order to increase his or her probability of being promoted in the future. They go on to show that this approach is consistent with empirical evidence concerning both the firm-size-wage effect and inter-industry wage differences.

Now we turn to our approach and contribution. Our approach is similar to that in Zabojnik and Bernhardt (2001) in that it is the signaling aspect of promotions that results in incentives associated with promotion. In our model each firm consists of a low-level job and a high-level job, where output increases with ability more quickly in the high-level job. We assume workers are in the labor market for two periods, where workers are more productive in the second period because of the accumulation of both general and firm-specific human capital. Also, at the beginning of the second period, because some information about a worker’s ability is only directly revealed to the firm employing the worker, promotions serve as signals of worker ability. The final key assumption is that output is a function of ability, effort, and the accumulation of human capital, with the result that a young worker can positively affect the current employer’s subsequent beliefs about his or her ability by increasing his or her effort level.

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3 Studies that find empirical support for the idea that promotions are associated with large wage increases include Gerhart and Milkovich (1989), Lazear (1992), Baker, Gibbs, and Holmstrom (1994a,b), and McCue (1996).

4 Other papers that investigate labor-market tournaments include Green and Stokey (1983), Nalebuff and Stiglitz (1983), Mookherjee (1984), and Rosen (1986).
We first conduct two preliminary analyses of this model. In the first, firms are restricted to employing standard promotion practices. In the second, firms are restricted to up-or-out contracts. In each analysis we find results related to Holmstrom (1982). Holmstrom investigated a related model with symmetric learning and a single job. He found that, because high effort now improves subsequent beliefs of all firms about a worker’s ability, workers have an incentive to exert high effort early in their careers in order to improve those subsequent beliefs and thus earn higher wages. Each of our preliminary analyses relies on an asymmetric-learning variant of Holmstrom’s argument. In our analysis a young worker provides high effort in order to improve the belief of the worker’s current employer concerning the worker’s ability. The return to improving this belief is that it increases the probability the worker will be promoted in the following period and, in turn, receive a higher wage because the positive signal associated with the promotion increases other firms’ willingness-to-pay for the worker’s services.

We then turn to the main analysis in which firms are allowed to choose between standard promotion practices and up-or-out contracts, where as indicated above our focus is on identifying the factors that affect which type of promotion practice is adopted. We find a number of factors determine the choice between standard promotion practices and up-or-out contracts. First, our main result is that a low level of firm-specific human capital results in up-or-out contracts while a high level results in standard promotion practices. Second, we find that in our basic model as the two jobs become more similar either contract can be preferred, where which is preferred depends on which job an average ability old worker is more productive. Third, we also find that, if similar to Lazear and Rosen (1981) firms can commit to a wage floor for workers who are promoted, then up-or-out contracts are preferred when the jobs are similar and effort provision is important.

There are a number of reasons why up-or-out contracts are more likely when firm-specific human capital is low. One reason is straightforward. With up-or-out contracts any worker who is not promoted must leave the firm, i.e., more workers lose the value associated with firm-specific human capital given up-or-out contracts than given standard promotion practices. In turn, when firm-specific human capital is low this cost of employing up-or-out contracts is low so the disadvantage of up-or-out contracts is small. Another reason concerns incentives for effort. Suppose there is a small amount of firm-specific human capital and standard promotion practices are employed. Then the extra wage associated with a promotion is frequently larger than the extra output associated with a promotion, so few workers are promoted and the incentive for young workers to provide effort is low. In contrast, since with up-or-out contracts an old
worker can only be kept by promoting the worker, when there is little firm-specific human capital up-or-out contracts result in many more promotions and much higher incentives. 

The reasons standard promotion practices are typically employed when firm-specific human capital is large are as follows. When firm-specific human capital is large, the cost of misallocating an old worker becomes large. The result is that the misassignment of old workers to jobs due to signaling under standard promotion practices becomes small, while the probability of turnover under up-or-out contracts also gets small. But this favors standard promotion practices for two reasons. First, the assignment of workers to jobs under standard promotion practices is more efficient. Second, as the probability of turnover falls to zero under up-or-out contracts the effort level of young workers under up-or-out falls to its minimum level.

In terms of our analysis of the similarity of jobs, we believe our most interesting result concerns what happens when we allow firms when hiring young workers to commit to a wage floor for old workers assigned to the high-level job in the following period. With this extra assumption we find that up-or-out contracts are preferred if the two jobs are very similar and effort provision is important. The logic here is as follows. On the one hand, even with the ability to commit to a wage floor for promoted workers the outcome given standard promotion practices when jobs are very similar is poor. That is, even with a wage floor, if jobs are very similar standard promotion practices result in no promotions due to the signaling aspect of promotion and young worker effort is at its minimum level because there are no promotions. On the other hand, suppose jobs are very similar and firms employ up-or-out contracts. Then the ability to commit to a wage floor means firms can achieve a positive probability of turnover which, in turn, increases young-worker effort above its minimum level. In turn, as long as effort provision is sufficiently important, the higher young-worker effort under up-or-out contracts means up-or-out is preferred.

We feel these results are of interest because they match up well with where we typically see up-or-out contracts in real-world settings. Consider, for example, the setting characterized by up-or-out contracts which most of us know best which is academic employment. Before and after the tenure decision the nature of the job is pretty much the same in that in both cases the job consists of research, teaching, and administrative work (although the mix typically changes somewhat upon receiving tenure). In addition, although there is some firm-specific human capital such as the human capital associated with collaborating with specific coauthors who might be more difficult to work with after a move, we argue
that for most faculty members this type of human capital is quite limited. Hence, the fact that academia is typically characterized by up-or-out contracts is consistent with our theoretical predictions. In Section IV we discuss in more detail the extent to which academia is consistent with our theoretical findings, and also discuss the extent to which our approach is consistent with evidence concerning large law firms.

The outline for the paper is as follows. Section II presents the model and provides preliminary analyses concerning what happens given standard promotion practices only and what happens given up-or-out only. Section III presents our main analysis which concerns the choice between standard promotion practices and up-or-out contracts. Section IV first compares our results concerning when up-or-out is likely to be employed with available evidence on the subject and then discusses alternative explanations for the use of up-or-out. Section V presents concluding remarks.

II. MODEL AND PRELIMINARY ANALYSIS

In this section we present our model and two preliminary analyses which concern what happens when firms are restricted to standard promotion practices only and what happens when they are restricted to up-or-out contracts only. The following section presents the main analysis which identifies the factors that determine the choice of standard promotion practices versus up-or-out contracts.

A) The Model

There is free entry into production, where all firms are identical and the only input is labor. A worker’s career lasts two periods, where in each period labor supply is fixed at one unit for each worker. We call workers in their first period in the labor market young and those in their second period old. Worker i’s innate ability is denoted $\theta_i$ and can be either high or low, i.e., $\theta_i \in \{\theta_L, \theta_H\}$ and $\theta_H > \theta_L > 0$. At the beginning of a worker’s career, all the firms and the worker himself know that the worker is of innate ability $\theta_H$ with probability $p$ and $\theta_L$ with probability $1-p$, where $\theta' = p\theta_H + (1-p)\theta_L$. A worker’s effective ability is a function of the worker’s innate ability and the worker’s labor-market experience. Letting $t$
denote calendar time, we assume that worker i’s effective ability in period t, \( \eta_{it} \), is given by \( \eta_{it} = \theta_i \) when the worker is young and \( \eta_{it} = \theta_{ik}, k > 1 \), when the worker is old.

A firm consists of two jobs, denoted 1 and 2. If young worker i is assigned to job j in period t, then the worker produces

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y_{ijt} = [d_j + c_j(\eta_{it} + e_{it} + \varepsilon_{it})],
\]

while old worker i assigned to job j in period t produces

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y_{ijt} = (1 + s_{it})[d_j + c_j(\eta_{it} + e_{it} + \varepsilon_{it})].
\]

In (1) and (2) \( d_j \) and \( c_j \) are constants known to all labor-market participants, \( e_{it} \) is worker i’s effort choice in period t, \( \varepsilon_{it} \) is a noise term drawn from a normal distribution with mean zero and variance \( \sigma^2 \), and \( s_{it} \) equals s, s > 0, if the worker was employed at the same firm in the previous period and zero otherwise. A worker chooses an effort level anywhere in the interval \([e_L, e_H]\), where we assume workers derive disutility from effort. Specifically, \( \alpha g(e_{it}) \) is worker i’s disutility from effort in period t, where \( g'(e) = 0, g'(e) > 0, g''(e) > 0 \) for all \( e > e_L \), and \( \alpha \) is a parameter that captures the importance of worker effort (the smaller is \( \alpha \) the more important is worker effort).\(^6\) Note that in this specification \( e_L \) can be thought of as a sufficiently low effort level that generates no disutility, or as a minimum effort level that can be enforced by contract. We also assume that \( e_{it} \) is above the first-best effort level for young workers, but sufficiently low that it is more efficient for young workers to choose the maximum effort level rather than the minimum effort level, i.e., given later assumptions the latter condition reduces to \( c_1 e_{it} - \alpha g(e_{it}) > c_1 e_L - \alpha g(e_L) \).

Let \( \eta'(e_1, e_2) \) denote the effective ability level at which a worker who chooses effort level \( e_1 \) if assigned to job 1 and effort level \( e_2 \) if assigned to job 2 yields the same expected net surplus at the two jobs. That is, \( \eta'(e_1, e_2) \) solves \( d_1 + c_1(\eta'(e_1, e_2) + e_1) - \alpha g(e_1) = d_2 + c_2(\eta'(e_1, e_2) + e_2) - \alpha g(e_2) \). We assume \( e_2 > c_1 > 0 \) and \( d_2 < d_1 \), i.e., as in Rosen (1982) output increases more quickly with ability in the higher-level job. Thus, given full information about worker abilities and effort choices, the efficient assignment rule for period t is to assign worker i to job 1 if \( \eta_{it} < \eta'(e_{i1}, e_{i2}) \) and to job 2 if \( \eta_{it} > \eta'(e_{i1}, e_{i2}) \), where \( e_{ijt} \) is individual i’s effort choice in period t if assigned to job j. Note that one reason this is the efficient assignment rule is that in our specification there is no difference in the rate of learning across jobs.

\(^6\) We also assume that \( \alpha g(e_L) \) is sufficiently small that working is always equilibrium behavior and that \( g(e) \) is sufficiently convex that under both up-or-out contracts and standard promotion practices there is a unique young-worker-effort level in equilibrium.
We assume that a worker’s output in any period is privately observed by the worker’s employer in that period. In addition to this private information, we assume that the job assignment offered to an old worker by the worker’s previous employer is public information. The result is that a promotion at the beginning of a worker’s second period in the labor force serves as a signal of the worker’s ability. The other important information assumption is that a worker’s effort choice in any period is privately known by the worker.

We assume that workers and firms are risk neutral and have a zero rate of discount. Additionally, there is no cost to workers from changing firms or to firms from hiring or firing workers. Given these assumptions, there is typically no benefit to long-term contracts so throughout most of our analysis we assume that wages are determined by spot-market contracting. Note further that, since a worker’s output is privately observed by the worker’s employer rather than being publicly observed and verifiable, any wage specified in such a spot contract consists of a wage determined prior to production rather than a wage determined by a piece-rate contract where compensation depends on the realization of output.

The wage setting process and timing of events is similar to that assumed in Zabojnik and Bernhardt (2001). At the beginning of each period, each firm chooses whether to offer standard promotion practices or up-or-out contracts to young workers and then offers each old worker that it employed in the previous period a job assignment for that period or fires the worker, where these decisions are publicly observed. For the job assignment/firing decision we assume a firm does not retain any worker it anticipates leaving with probability one during the wage-determination process. This assumption is consistent with the existence of a small cost of retaining a worker who then chooses to leave. Following Greenwald (1986), Lazear (1986), and Milgrom and Oster (1987), we then assume that the initial employer can make counteroffers. That is, after the initial stage, all firms simultaneously offer each worker in the economy a wage for that period. Then each firm makes a wage counteroffer to each old worker it employed in the previous period. Each worker then chooses to work at the firm from which he or she anticipates receiving the highest expected utility, where for young workers this means lifetime expected utility (note, utility in any period just equals the wage received minus the disutility from

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7 The one exception is that a firm does not make initial wage offers or counteroffers to the workers it fired at the beginning of the period. Also, when a firm makes an offer to an old worker it did not employ in the previous period, the firm does not base the offer on how many old workers the worker’s original employer fired, retained, or assigned to each job in the current period. This could be derived endogenously by assuming each firm is restricted to hiring at most one young worker each period.
effort). If there are multiple firms tied at this highest expected utility, the worker chooses randomly among these firms unless one of these was the worker’s employer in the previous period, in which case the worker remains with that firm. This tie-breaking rule is equivalent to assuming an infinitesimally small moving cost. At the end of each period firms privately observe each of its worker’s outputs.

To reduce the number of cases that need to be considered, we restrict the analysis to parameterizations that satisfy the following two conditions. The first condition is that $d_1+c_1(\theta_H+e_H)>d_2+c_2(\theta_H+e_H)$. This condition ensures that all young workers are assigned to the low-level job. The second condition is that $\theta_L<\eta'(e_L,e_L)<\theta_H$. This assumption ensures that in our asymmetric learning equilibrium it is efficient for some but not all workers to be promoted when they become old.

Finally, we limit attention to Perfect Bayesian Equilibria in which beliefs concerning off-the-equilibrium path actions are consistent with each such action being taken by the type with the smallest cost of choosing that action. Restricting attention in this way means our analysis is characterized by a winner’s curse result similar to that found in the related analysis of Milgrom and Oster (1987), i.e., firms other than an old worker’s initial employer are only willing to offer the worker a wage equal to the expected productivity at such a firm of the lowest ability worker who has the same labor market signal (meaning job assignment).8

B) Analysis Given Standard Promotion Practices

In this subsection we present our first preliminary analysis that considers the model of the previous subsection assuming firms only employ standard promotion practices. By this we mean that when a firm hires a young worker in period $t$ it does not restrict its behavior concerning how the worker will be treated by the firm in period $t+1$. Hence, at the beginning of period $t+1$ each such worker is either assigned to job 1, assigned to job 2, or fired. In the next subsection we consider what happens when firms only offer up-or-out contracts, where a worker hired under an up-or-out contract cannot be assigned to job 1 by his or her initial employer when the worker becomes old.

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8 In our model this is not an immediate implication of restricting the analysis to Perfect Bayesian Equilibria both because of the presence of firm-specific human capital which Milgrom and Oster did not incorporate into their analysis, and because in our model a promoted worker is signaled to be of high ability while in theirs a promoted worker’s ability becomes public knowledge. Note that our assumption concerning off-the-equilibrium path actions is basically equivalent to the notion of a Proper Equilibrium first discussed in Myerson (1978).
We briefly describe a benchmark analysis that concerns what happens given standard promotion practices if effort and output are both publicly observable (but only effort is verifiable). In this case at every date, given the information available, workers choose efficient effort levels, are assigned to jobs in the efficient fashion, and switch employers in the efficient fashion. To be more specific, the equilibrium is described by the following. First, equilibrium effort equates the marginal disutility of effort with the marginal productivity of effort. For example, if young worker i is assigned to job 1 in period t, then $e_{it}$ satisfies $\alpha g'(e_{it}) = c_1$. Second, because of the presence of firm-specific human capital, there is no turnover. Third, a worker is always assigned to the job that maximizes the worker’s expected output minus his or her disutility from effort. For example, given our parameter restriction $d_1 + c_1(\theta_H + e_H) > d_2 + c_2(\theta_H + e_H)$, every young worker is assigned to job 1. Fourth, the young-worker wage satisfies a zero-profit condition, while the wage determination process yields that each old worker is paid an amount equal to the worker’s expected productivity at an alternative employer.\(^9\)

The main lesson of the benchmark analysis is that, if effort and output are both publicly observable, then effort choices, job assignments, and turnover decisions are efficient. As we show below, in contrast, once a worker’s effort choice is privately known by the worker and a worker’s output is privately observed by the worker’s employer, then various inefficiencies arise. For example, the effort choices of old workers are too low due to moral hazard, while as in Holmstrom (1982) young-worker effort will typically not be efficient. Another inefficiency concerns the assignment of old workers to jobs. That is, as was initially found in Waldman (1984), firms assign too few old workers to the high-level job in order to avoid sending the positive signal about ability associated with assignment to the high-level job.

Suppose that a worker’s effort choice each period is privately known by the worker, while the worker’s output each period is privately observed by the worker’s employer. We start with some preliminary results. First, as in the benchmark case, our parameter restriction $d_1 + c_1(\theta_H + e_H) > d_2 + c_2(\theta_H + e_H)$ yields that all young workers are assigned to the low-level job. Second, the wage paid to young workers, denoted $w_Y$, is above expected output and is such that a firm hiring a young worker earns zero expected profits from the hire. The basic logic here is that both because of information rents and firm-specific human capital old workers are paid less than their expected output, so the zero-expected-profit condition

\(^9\) To be precise, this is the old worker wage if the worker’s effort level would be the same across the two firms. If the effort levels would be different, then the wage needs to adjust for the different disutilities of effort.
requires that young workers are paid more than their expected output. Third, young-worker effort, denoted $e_Y^S$, will not typically be efficient, where the logic for this result is described below. Fourth, since effort is not observable and workers only live for two periods, an old worker will expend the minimum effort level.

We now formally state what happens in this case. Below $\eta_i(y_{i1t-1})$ is worker i’s expected effective ability in period t as a function of the realization of $y_{i1t-1}$, $w^O_\alpha(y_{i1t-1})$ is the wage paid to old worker i in period t as a function of the realization of $y_{i1t-1}$, $J_\alpha$ is the firm that individual i works at in period t, and $e_Y^*$ is the first-best-efficient effort level for a young worker assigned to job 1, i.e.,

$$\alpha^g(e_Y^*)=c_1.$$

**Proposition 1**: If firms employ standard promotion practices, then there exists a value $\eta^S_\alpha$, such that i) through iii) describe equilibrium behavior in each period t.

i) All young workers are assigned to job 1, choose effort $e_Y^S$, and are paid $w_Y^S$, $w_Y^S>d_1+c_1(\theta'+e_Y^S)$, where $e_Y^S$ can be greater than, less than, or equal to $e_Y^*$.

ii) If old worker i is such that $\eta_i(y_{i1t-1}) \geq \eta^S_\alpha$, then the worker is promoted, remains at firm $J_{i1t-1}$, chooses effort level $e_L$, and is paid $w^O_\alpha(y_{i1t-1})=d_2+c_2(\theta^L+k+e_L)$.

iii) If old worker i is such that $\eta_i(y_{i1t-1}) < \eta^S_\alpha$, then the worker is not promoted, remains at firm $J_{i1t-1}$, chooses effort level $e_L$, and is paid $w^O_\alpha(y_{i1t-1})=d_1+c_1(\theta_L+k+e_L)$.

**Proposition 1** tells us that, if a worker’s effort is privately known by the worker, a worker’s output is privately observed by the worker’s employer, and firms employ standard promotion practices, then there is a critical value for $\eta^S_\alpha(y_{i1t-1})$, that determines what happens to the worker when he or she becomes old. If $\eta^S_\alpha$ is above the critical value, then the worker is promoted and stays with the initial employer. If $\eta^S_\alpha$ is below the critical value, then the worker is not promoted but again stays with the initial employer. Further, in each case the wage paid to a worker equals the productivity at another potential employer of the lowest ability worker with the same labor-market signal. For example, the wage

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10 In the standard-promotion-practices case we focus on the unique equilibrium characterized by no workers being fired. This is the only equilibrium if s is sufficiently large or the two jobs are sufficiently similar. We prove the former in the proof of Proposition 1. Also, to simplify descriptions of behavior, throughout the paper we assume that an old worker is assigned to job 2 by the worker’s current employer whenever the firm is indifferent between assigning the worker to jobs 1 and 2.
paid to a worker not promoted equals the productivity of an old worker with innate ability $\theta_L$ at an alternative employer. The logic here is that, given our assumption that an old worker’s previous employer can make a counteroffer and given our assumption about beliefs concerning off-the-equilibrium path moves, there is a winner’s curse problem which means that other potential employers are only willing to pay the lowest possible productivity of a worker with the same labor-market signal (see Greenwald (1986) and Milgrom and Oster (1987) for earlier analyses characterized by this winner’s curse result). Hence, in order to retain a worker, this is all an old worker’s previous employer will offer.

In addition to these results concerning the existence of a critical value for $\eta^e_{it}$ and how wages are determined, there are a number of interesting aspects of this proposition. The first is that, since $\eta^{S+} > \eta^*(e_{L}, e_{L})$, we have that fewer workers are assigned to the high-level job than is efficient given the initial employer’s knowledge concerning the worker’s effective ability. The logic here is the same as that found in a number of papers in which promotion to a high-level job serves as a signal. Because assigning an old worker to the high-level job rather than the low-level job sends a signal that the worker is of high ability, firms give promoted workers large wage increases in order to stop them from being bid away. In turn, because of the need to pay a promoted worker a high wage, a worker’s initial employer will only assign the worker to the high-level job if his or her productivity in the high-level job significantly exceeds productivity in the low-level job.

The other interesting aspect of this case is the effort choice of young workers. As stated in the proposition, this effort choice can be either above, below, or equal to the first-best effort level. The logic here combines the arguments in Holmstrom (1982) and Zabojnik and Bernhardt (2001). The return to a young worker exerting effort is that increasing effort increases output and thus improves the employer’s belief in the following period concerning the worker’s effective ability. In turn, if this belief is pushed across the critical value $\eta^*_{it}$, then the worker benefits because he or she is promoted and receives the higher wage due to the promotion. Since there is nothing in this mechanism that ties the effort choice to the first-best level, depending on the parameterization equilibrium effort can be above, below, or equal to the first-best level. In Section III we discuss how this effort choice varies with some of the exogenous parameters.
C) Analysis Given Up-or-Out Contracts

This subsection presents our second preliminary analysis that considers the model under the assumption firms only offer up-or-out contracts. Just as a reminder, a worker hired under an up-or-out contract must either be promoted to job 2 or fired by his or her initial employer when the worker becomes old. Similar to the previous analysis, we begin by briefly describing a benchmark analysis that considers what happens given up-or-out contracts if effort and output are both publicly observable (but only effort is verifiable). We then consider what happens when a worker’s effort is privately known by the worker and a worker’s output is privately observed by the worker’s current employer.

The benchmark equilibrium is now described as follows. First, as in the previous case, effort levels are efficient in that equilibrium effort equates marginal disutility of effort with the marginal productivity of effort. Second, as opposed to the previous benchmark analysis, because of the restriction that only promoted workers can be retained, there is now turnover. In particular, there is a critical value for expected effective ability such that an old worker whose expected effective ability is greater than this value is retained and promoted, while a worker whose expected effective ability is less than this value is fired and assigned to job 1 by his or her new employer. Third, as in the previous benchmark analysis, the young-worker wage satisfies a zero-profit condition while each old worker is paid an amount equal to expected productivity at a firm other than the worker’s employer in the previous period.\(^\text{11}\)

Taken together, our two benchmark analyses tell us that, given publicly observable and verifiable effort and given publicly observable output, there is no role for up-or-out contracts in our model. To see this, let us briefly compare the two benchmark analyses. In the benchmark analysis with standard promotion practices, job assignments, effort levels, and turnover decisions are all efficient. In contrast, in the benchmark analysis with up-or-out contracts, effort levels are efficient but turnover and job-assignment decisions are not.\(^\text{12}\) For example, because of the presence of firm-specific human capital efficiency requires all old workers remain with their initial employers, but the restriction that an old worker’s initial employer cannot assign the worker to job 1 causes the firm to sometimes fire the worker. Also, because a firm can only keep an old worker by assigning the worker to job 2, more workers are

\(^{11}\) Similar to footnote 9, this is the old worker wage for workers who are fired and those who are retained for whom equilibrium effort is the same whether the worker stays or moves. If in the latter case the effort levels are different, then the wage needs to adjust for the different disutilities of effort.

\(^{12}\) Turnover and job assignments are efficient from a second-best perspective, i.e., they are efficient given the restriction that an old worker can only be assigned to job 1 if he or she changes employers.
assigned to job 2 than is efficient. In turn, the inefficient turnover and job-assignment decisions associated with up-or-out contracts mean that a young worker’s expected lifetime utility is higher in the benchmark analysis of the previous subsection rather than the benchmark analysis here. In other words, if firms could offer either standard promotion practices or up-or-out contracts, all young workers in equilibrium would choose firms offering standard promotion practices. In the next section we show this is not the case when worker effort and output are not publicly observable.

We now turn to our analysis of what happens given only up-or-out contracts are offered and a worker’s effort is privately known by the worker while a worker’s output is privately observed by the worker’s current employer. There are a number of preliminary results here that are the same as preliminary results in the previous section given these information assumptions and that firms only offer standard promotion practices. First, all young workers are assigned to the low-level job. Second, the wage paid to young workers, denoted $w_Y^U$, is above a young worker’s expected output. Third, young-worker effort, denoted $e_Y^U$, will not typically be efficient. Fourth, every old worker chooses the minimum effort level $e_L$.

We now formally state what happens in this case. Below $w_O^U(y_{it-1})$ is the wage paid to old worker $i$ in period $t$ as a function of the realization of $y_{it-1}$.

Proposition 2: If firms employ up-or-out contracts, then there exists a value $\eta_{it}^{U+}$, $\eta_{it}^{U+}<\eta'(e_L,e_L)<\eta_{it}^{S+}$, such that i) through iii) describe equilibrium behavior in each period $t$.

i) All young workers are assigned to job 1, choose effort $e_Y^U$, and are paid $w_Y^U$, $w_Y^U>d_1+c_1(\theta'+e_Y^U)$, where $e_Y^U$ can be greater than, less than, or equal to $e_Y^*$. 

ii) If old worker $i$ is such that $\eta_{it}(y_{it-1}) \geq \eta_{it}^{U+}$, then the worker is promoted, remains at firm $j_{it-1}$, chooses effort level $e_L$, and is paid $w_O^U(y_{it-1})=d_1+c_1(\eta_{it}^{U+}+e_L)$.

iii) If old worker $i$ is such that $\eta_{it}(y_{it-1}) < \eta_{it}^{U+}$, then the worker is fired, is assigned to job 1 by the new employer, chooses effort level $e_{it}$, and is paid $w_O^U(y_{it-1})=d_1+c_1(E(\theta)k+e_{it})$, where $E(\theta)$ is the expected innate ability of a worker who is fired.

Proposition 2 tells us that, if a worker’s effort is privately known by the worker, output is privately observed by the worker’s employer, and firms employ up-or-out contracts, then there is a critical value for $\eta_{it}(y_{it-1})$, $\eta_{it}^{U+}$, that determines what happens to the worker when he or she becomes old. If $\eta_{it}$ is
above the critical value then the worker is promoted and stays with the initial employer, while if \( \eta^e \) is below the critical value then the worker is fired and is assigned to the low-level job by the new employer. In other words, the difference between the equilibrium outcome here and that described in Proposition 1 is that, because the firm is contractually obligated to only retain promoted workers, for sufficiently low values for \( \eta^e \) the worker is fired rather than retained and the new employer then assigns the worker to the low-level job.

One interesting aspect of Proposition 2 is that \( \eta^{U+} < \eta'(e_L, e_L)^+ < \eta^S+ \). This result states that, if anything, too many old workers are assigned to the high-level job. This stands in contrast to the result we found in Proposition 1 for standard promotion practices where too few old workers were assigned to the high-level job. The logic is as follows. Given standard promotion practices, too few old workers are assigned to the high-level job because firms have an incentive to avoid the positive signal and corresponding high wage associated with a promotion. With up-or-out contracts, however, the only way to avoid the positive signal and high wage is by firing the worker. But if the worker is fired the firm earns zero profits from the worker. The result is that the firm retains and promotes every old worker for whom it earns positive expected profits and this, in turn, yields \( \eta^{U+} < \eta'(e_L, e_L) \). Note also that this result explains why the wage for promoted workers is \( d_1 + c_1(\eta^{U+} + e_L) \) rather than \( d_2 + c_2(\eta^{U+} + e_L) \). That is, given \( \eta^{U+} < \eta'(e_L, e_L) \), it is more efficient for another firm hiring a worker whose expected effective ability is \( \eta^{U+} \) to assign the worker to job 1 rather than job 2.

Note further that one interpretation of the finding \( \eta^{U+} < \eta'(e_L, e_L) \) is that the firm retains and promotes too many workers, i.e., the firm retains and promotes some workers who would be more efficiently assigned to job 1. However, this is not our preferred interpretation. The above interpretation focuses on the idea that the actual critical value, \( \eta^{U+} \), is lower than the efficient critical value given equilibrium effort levels, \( \eta'(e_L, e_L) \). Our preferred interpretation takes into account the second-best nature of the problem. In this setting a firm can only assign a retained old worker to job 2, and an old worker can thus only be assigned to job 1 by switching employers and losing the value of the worker’s accumulated firm-specific human capital. Thus, since \( \eta^{U+} \) satisfies \((1+s)[d_2 + c_2(\eta^{U+} + e_L)] = d_1 + c_1(\eta^{U+} + e_L)\), it is the optimal critical value given this constraint.\(^{13}\)

\(^{13}\) \( \eta^{U+} \) is defined by this equation as long as \( \eta^{U+} > \theta_k \). As discussed in the next section, for some parameterizations \( \eta^{U+} = \theta_k \) in which case the condition \((1+s)[d_2 + c_2(\eta^{U+} + e_L)] \geq d_1 + c_1(\eta^{U+} + e_L)\) must be satisfied. However, even in this case \( \eta^{U+} \) is the optimal critical value given the constraint that a worker can only be retained if the worker is assigned to job 2.
Another interesting aspect of this case is the effort choice of young workers. This effort choice is determined by a logic similar but not identical to that which determined young-worker effort given standard promotion practices. As in the case of standard promotion practices, by increasing his or her effort level a young worker increases output produced, and this improves the employer’s belief in the following period concerning the worker’s expected effective ability. In turn, if this belief is pushed across the critical value $\eta^{U+}$, then the worker benefits because he or she is retained and promoted and receives the higher wage that corresponds to retention. Note that, as indicated, the logic here is not identical to that in the standard promotion practices case since here an old worker whose value for $\eta_e$ is below the relevant critical value is fired, while in the standard promotion practices case the worker is retained and assigned to job 1.

As a final point, under both contractual forms old workers systematically provide the minimum effort level $e_L$, where in each case this is due to the interaction between moral hazard on the part of workers and no future periods in which workers can be compensated for higher current effort. Clearly, from a real-world perspective this result is a bit unrealistic. We could avoid the result by giving firms an alternative way of providing incentives such as increased supervision or a higher reliance on incentive pay. Under some parameterizations we would then find that, because of the incentives provided through future promotion, the alternative way of providing incentives would be used more heavily with older workers (see Gibbons and Murphy (1992) for a theoretical and empirical analysis along this line). However, given this issue is not the focus of the current paper, we have decided to stay with the current specification which is more tractable but somewhat unrealistic concerning the effort levels of old workers.

III. CHOOSING A CONTRACTUAL FORM

In this section we explore the factors that serve to determine the choice of standard promotion practices versus up-or-out contracts. Our focus is on two aspects of the model – the degree of firm-specific human capital in the economy and the similarity of the two jobs. For each factor we first derive comparative static results and then derive resulting implications for the choice of contractual form.

We begin by considering the role of firm-specific human capital. Suppose first that firms employ standard promotion practices and there is a decrease in firm-specific human capital, i.e., a decrease in $s$. There are two important effects. The first is an increase in the misassignment of old workers across jobs, i.e., $\eta^{S+}$ rises, or equivalently, $\eta^{S+}-\eta'(e_L,e_L)$ rises. Further, for $s$ sufficiently small no one is promoted, i.e.,
\( \eta^{S+} = \theta_{Hk} \)\(^{14}\). The logic here is that, as \( s \) falls, the cost in expected lost productivity due to misassigning an old worker of ability \( \eta_{it} > \eta'_{(eL, eL)} \), \( (1+s)[(c_2-c_1)\eta_{it}^{e} - \eta'_{(eL, eL)}] \), falls and this, in turn, causes the critical value \( \eta^{S+} \) to rise. Additionally, given the wage setting process we assume, for any fixed value of \( \eta^{S+} \) and \( s \) sufficiently small the extra productivity associated with promoting an \( \eta^{S+} \) worker must be below the extra wage due to signaling associated with promoting the worker. The result is that, for \( s \) sufficiently small, no one is promoted.

The second important effect is that the effort level of young workers, \( e_{Y}^{S} \), typically falls as \( s \) becomes smaller, where for \( s \) sufficiently small \( e_{Y}^{S} \) equals the minimum effort level \( e_{L} \). The logic here follows from the first effect just discussed. That is, as \( s \) falls the probability of earning a promotion falls, and this lowered probability of promotion typically reduces the incentive for young workers to provide effort and thus results in a lower equilibrium effort level. Further, since for \( s \) sufficiently small the probability of promotion equals zero, this same logic tells us that \( e_{Y}^{S} \) must equal \( e_{L} \) for very small \( s \).

Now suppose firms employ up-or-out contracts and there is a decrease in firm-specific human capital. The key here is that the loss in productivity due to unused firm-specific human capital when a worker is terminated falls. This, in turn, translates into firms having a smaller incentive to retain their old workers so the critical value \( \eta^{U+} \) rises. In terms of this condition, as \( s \) approaches zero the critical value \( \eta^{U+} \) approaches \( \eta'_{(eL, eL)} \) (rather than going to \( \theta_{Lk} \) as was the case with \( \eta^{S+} \) in the standard promotion practices case), while for \( s \) sufficiently large \( \eta^{U+} \) equals \( \theta_{Lk} \), i.e., all workers are retained. To see this, note that if \( \eta^{U+} > \theta_{Lk} \) then \( \eta^{U+} \) is defined by the condition \( (1+s)[(d_2+c_2(\eta^{U+} + e_{L})] = d_1 + c_1(\eta^{U+} + e_{L}) \), where this equation reduces to the condition that defines \( \eta'_{(eL, eL)} \) when \( s \) equals zero while according to this equation it falls without bound as \( s \) grows. Note further that, since the probability of retention and promotion does not fall to zero as \( s \) goes to zero, \( e_{Y}^{U} \) remains strictly above \( e_{L} \) as \( s \) falls to zero. However, since the probability of retention goes to one as \( s \) gets sufficiently large, \( e_{Y}^{U} \) equals \( e_{L} \) for \( s \) sufficiently large.

We summarize the above discussion in Lemma 1.

**Lemma 1**: Holding all other parameters fixed, if only standard promotion practices are employed, then there exists a highest value \( s^{Sr} \), \( s^{Sr} > 0 \), such that \( \eta^{S+} = \theta_{Hk} \) for \( s \leq s^{Sr} \) and \( \eta^{S+} \) is decreasing in \( s \) for \( s > s^{Sr} \).

\(^{14}\) Throughout the analysis we say a cutoff value equals \( \theta_{Hk} \) when no one is promoted and equals \( \theta_{Lk} \) when everyone is promoted.
Further, as $s$ approaches $s^r$ from above, $\eta^s$ approaches $\theta_l$, and $e_Y^s$ approaches $e_L$, while $e_Y^u = e_L$ for $s \leq s^r$.

Holding all other parameters fixed, if only up-or-out contracts are employed, then there exists a smallest value $s^u$, $s^u > 0$, such that $\eta^u = \theta_l$ for $s \geq s^u$ and $\eta^u$ is decreasing in $s$ for $s < s^u$. Also, as $s$ approaches $s^u$ from below $\eta^u$ approaches $\theta_l$ and $e_Y^u$ approaches $e_L$, while $e_Y^u = e_L$ for $s \geq s^u$.

We now turn to how the degree of firm-specific human capital in the economy affects the choice of contractual form. In particular, suppose that each firm has the option of employing either standard promotion practices or up-or-out contracts. The above analysis of how variation in $s$ affects equilibrium behavior given each type of contract tells us that up-or-out contracts will be preferred if $s$ is sufficiently small. There are a number of reasons for this result. As $s$ becomes small and given standard promotion practices, the misallocation of workers to jobs rises while young worker effort falls and eventually equals the minimum value. Second, as $s$ becomes small and given up-or-out contracts, the loss due to the inefficient allocation of old workers across firms falls and, in particular, falls towards zero as $s$ approaches zero. Finally, in contrast to what is true with standard promotion practices, as $s$ approaches zero and given up-or-out contracts, the assignment of workers across jobs moves towards the efficient assignment and young-worker effort remains strictly above $e_L$.

It is also the case that, if $s$ is sufficiently large, then standard promotion practices will be preferred. The logic here is as follows. With standard promotion practices there is no turnover and thus no loss in productivity due to inefficient turnover and the associated unused firm-specific human capital. Also, the misallocation of workers to jobs gets small because the loss in productivity associated with misassignment rises. On the other hand, with up-or-out contracts as $s$ gets large there is no turnover but the result is many old workers are assigned to job 2 who would produce more if assigned to job 1 by the same employer. Also, as turnover goes to zero, the incentive for young workers to provide effort goes to zero which means $e_Y^u$ equals the minimum value $e_L$. All these factors point in the direction of standard promotion practices being preferred when $s$ is sufficiently large.

These results are formalized in Proposition 3.

**Proposition 3:** Holding all other parameters fixed, if both standard promotion practices and up-or-out contracts are available, then there exists a largest value $s^+$ and a smallest value $s^{++}$, $0 < s^{++} < \infty$, such that
all young workers choose firms that employ up-or-out contracts if \( s < s^+ \) and all young workers choose firms that employ standard promotion practices if \( s > s^+ \).\(^{15}\)

We now turn our attention to the similarity of the two jobs, where our focus is on what happens when the two jobs become very similar. We begin by considering this issue in our basic model and then consider what happens when firms can commit to a wage floor for promoted workers. Suppose first that in our basic model firms employ standard promotion practices and there is an increase in \( c_1 \) and decrease in \( d_1 \), holding all other parameters fixed, such that \( \eta'(e_L, e_L) \) is unchanged. That is, suppose the parameters associated with job 1 are changed so that the two jobs become more similar in terms of the incremental productivity associated with extra ability, i.e., \( c_2 - c_1 \) falls, but the second-best efficient assignment of old workers across jobs is unchanged. There are two important effects. The first is an increase in the misassignment of old workers across jobs, i.e., \( \eta^{S^+} - \eta'(e_L, e_L) \) rises, where for sufficiently small values for \( c_2 - c_1 \) the critical value \( \eta^{S^+} \) equals \( \theta_H k \). The logic here is that as \( c_1 \) rises and the two jobs become more similar the cost in lost production due to misassigning an old worker of expected ability \( \eta^{S^+} > \eta'(e_L, e_L) \), \( (1+s)[(c_2-c_1)(\eta^{S^+} - \eta'(e_L, e_L)) \), falls. Hence, the critical value, \( \eta^{S^+} \), above which workers are assigned to job 2 and below which job 1 will rise. Further, because for any worker as \( c_2 - c_1 \) approaches zero the lost productivity from misassignment approaches zero, the signaling cost of promotion means a firm will promote no one for \( c_2 - c_1 \) sufficiently small.

The second important effect is that the effort level of young workers, \( e_Y^{S^+} \), typically falls as \( c_2 - c_1 \) falls, where for \( c_2 - c_1 \) sufficiently small this effort level equals the minimum value \( e_L \). The logic here follows from the first effect just discussed. That is, as the two jobs become more similar the probability of receiving a promotion falls, and this lowered probability of promotion will typically reduce the incentive for young workers to provide effort and thus result in a lower equilibrium effort level. Further, since for \( c_2 - c_1 \) very small the probability of promotion equals zero, this same logic also explains why \( e_Y^{S^+} \) equals \( e_L \) for \( c_2 - c_1 \) sufficiently small.

\(^{15}\) Note that an important assumption for the proposition to hold is that there is a maximum effort level \( e_H \) and that \( c_1 e_H - \alpha g(e_H) > c_1 e_L - \alpha g(e_L) \). The basic idea here goes back to results in Propositions 1 and 2 in which equilibrium effort levels are potentially above the efficient level. The role of the assumption \( c_1 e_H - \alpha g(e_H) > c_1 e_L - \alpha g(e_L) \) is to ensure that even if young worker effort exceeds the efficient level it is still superior to the minimum effort level. That is, as long as this condition is satisfied, the increase in effort that occurs in moving from standard promotion practices to up-or-out contracts when \( s \) is small (or moving from up-or-out contracts to standard promotion practices when \( s \) is large) must be welfare enhancing.
Now suppose firms employ up-or-out contracts and there is an increase in $c_1$ and decrease in $d_1$ such that $\eta'(e_L,e_L)$ is unchanged. There are again two important effects. First, the critical value $\eta^U+$ falls, where for sufficiently small values for $c_2-c_1$ this critical value equals $\theta_1 k$, i.e., everyone is retained and promoted. The logic here is that, as $c_2-c_1$ falls and the two jobs become more similar, the presence of firm-specific human capital makes it efficient to retain and promote a higher proportion of old workers. Further, this logic yields that it is efficient to retain and promote with probability one for $c_2-c_1$ sufficiently small. Second, for $c_2-c_1$ small young worker effort is small, where for $c_2-c_1$ sufficiently small young worker effort equals $e_L$. The logic here is that when the probability of retention gets large the extra wage associated with being retained falls towards zero so the incentive for effort also falls towards zero. Further, for $c_2-c_1$ sufficiently small the probability of retention is one and there is no incremental wage associated with being retained. Clearly in this case young worker effort equals $e_L$.

We formally state the results for $c_2-c_1$ very small in Lemma 2. Note that below a normalized increase in $c_1$ refers to an increase in $c_1$ and decrease in $d_1$ such that $\eta'(e_L,e_L)$ is unchanged.

**Lemma 2**: Holding all other parameters fixed, if firms employ standard promotion practices and there is a normalized increase in $c_1$ such that $c_2-c_1$ is sufficiently small, then $\eta^S+=\theta_1 k$ and $e_Y^S=e_L$. Holding all other parameters fixed, if firms employ up-or-out contracts and there is a normalized increase in $c_1$ such that $c_2-c_1$ is sufficiently small, then $\eta^U+=\theta_1 k$ and $e_Y^U=e_L$.

We now turn our attention to how the choice of contractual form depends on the similarity of the two jobs. In particular, suppose that each firm has the option of employing either standard promotion practices or up-or-out contracts, and as in Lemma 2 above $c_1$ and $d_1$ vary so that $\eta'(e_L,e_L)$ is unchanged but $c_2-c_1$ becomes very small. The above analysis tells us that which type of contract is employed depends on which job an old worker of average ability is more productive. That is, standard promotion practices are employed if an old worker of average ability is more productive at job 1, while up-or-out contracts are employed if an old worker of average ability is more productive at job 2. The logic follows immediately from Lemma 2. As $c_2-c_1$ becomes very small, under standard promotion practices all old workers are retained, assigned to job 1, and $e_Y^S=e_L$, while under up-or-out contracts all old workers are retained, assigned to job 2, and $e_Y^U=e_L$. Clearly, which contract is more efficient depends on which job an average ability old worker is more productive.
Proposition 4: Holding all other parameters fixed, if there is a normalized increase in $c_1$ such that $c_2-c_1$ is sufficiently small, then standard promotion practices are employed when $(1+s)[d_1+c_1(\theta'k+e_L)]>(1+s)[d_2+c_2(\theta'k+e_L)]$ while up-or-out contracts are employed when $(1+s)[d_1+c_1(\theta'k+e_L)]<(1+s)[d_2+c_2(\theta'k+e_L)]$ (either type of contract can be equilibrium behavior when $(1+s)[d_1+c_1(\theta'k+e_L)]=(1+s)[d_2+c_2(\theta'k+e_L)]$).\(^{16}\)

In our last analysis we extend the above analysis concerning the similarity of the two jobs by introducing a wage floor for promoted workers. That is, in this analysis when a firm hires a young worker it can commit to a lower bound on the wage it will pay workers assigned to job 2 in the following period.\(^{17}\) The idea that a firm can commit to subsequent wages for promoted workers is similar to the approach taken in analyses such as Lazear and Rosen (1981) and the classic analysis of up-or-out contracts of Kahn and Huberman (1988) discussed in the next section. In contrast to those analyses, however, we assume the firm commits to a wage floor rather than a specific wage. We believe this is more realistic since the possibility of mutually beneficial renegotiation would turn a commitment to a wage into a commitment to a wage floor.

The question is, how is the above analysis altered when this change is introduced? Consider first what happens when all firms offer standard promotion practices. In this case the answer is that the ability to commit to a wage floor for promoted workers does not change the analysis. That is, if $c_2-c_1$ is sufficiently small, it is still the case that no old workers are promoted and as a result young-worker effort $e_Y$ equals the minimum effort level $e_L$. The reason is that it is the signaling aspect of promotion that results in no promotions, and introducing the ability to commit to a wage floor does not allow firms to lower high promotion wages due to signaling.

Now consider the case in which all firms offer up-or-out contracts. In this case the ability to commit to a wage floor for promoted workers changes the analysis in a substantial way. As described above, when $c_2-c_1$ is very small, in the absence of wage floors the probability of promotion equals one and young-worker effort equals the minimum value. By committing to a wage floor for promoted workers, \(^{16}\) Note that, if $c_1=c_2$, then either type of contract can be employed since $(1+s)[d_1+c_1(\theta'k+e_L)]=(1+s)[d_2+c_2(\theta'k+e_L)]$.

\(^{17}\) Introducing the ability to commit to a wage floor for promoted workers does not change the qualitative results found above concerning firm-specific human capital. That is, up-or-out contracts would still be preferred when there is a small amount of firm-specific human capital and standard promotion practices would still be preferred when the degree of firm-specific human capital is large.
however, a firm can stop this from occurring. The first step of the argument is that by committing to a wage floor above the wage offers of other firms the firm can reduce the probability of retention and promotion below one. In turn, this results in an increase in young-worker effort above its minimum value. Further, as long as effort provision is sufficiently important, i.e., $\alpha$ is sufficiently small, this will be the equilibrium outcome when firms can only offer up-or-out contracts.

We now translate these findings into what happens when firms can offer either standard promotion practices or up-or-out contracts and firms can commit to wage floors for promoted workers. If $\alpha$ is sufficiently small and $c_2-c_1$ is sufficiently small, then up-or-out contracts will be preferred. The logic for this result follows immediately from the above discussion. Given standard promotion practices, if $c_2-c_1$ is sufficiently small, then no one is promoted and young-worker effort equals $e_L$. On the other hand, given up-or-out contracts, these two parameter restrictions result in a probability of promotion less than one and young-worker effort above $e_L$. In turn, with $\alpha$ sufficiently small young worker effort is sufficiently important that the higher young-worker effort associated with up-or-out contracts must make up-or-out the preferred contract choice.

We formalize this discussion in Proposition 5.18

**Proposition 5:** Suppose firms can offer either standard promotion practices or up-or-out contracts, and firms can commit to wage floors for promoted workers. Holding all other parameters fixed, if $\alpha$ is sufficiently small and there is a normalized increase in $c_1$ such that $c_2-c_1$ is sufficiently small, then all young workers choose to work at firms that employ up-or-out contracts.

Note that the logic here is similar to the logic that appears in an informal discussion in Prendergast (1993). In his formal analysis, Prendergast considers what we are calling a standard-promotion-practices setting in which firms commit to a wage for promoted workers and workers invest in firm-specific human capital in order to increase their chances for promotion. In his setting, if jobs are similar, then the incentive for firms to promote workers falls and this reduces the incentive for young

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18 For this proposition we impose the extra assumption that the upper bound on effort, $e_H$, is “high.” See the Appendix for further discussion of this assumption. Note also that, in contrast to our original specification, it is no longer the case that the upper bound on effort necessarily exceeds the first-best-efficient effort level for young workers. This is because, as $\alpha$ falls to zero, the unconstrained first-best-efficient effort level for young workers goes to infinity.
workers to invest. Prendergast’s informal argument is that, when jobs are similar, firms will employ up-or-out contracts in order to increase the incentive for young workers to invest. His logic is that up-or-out contracts can be used to increase the probability an old worker will be assigned to the high-level job by eliminating the firm’s ability to assign old workers to the low-level job (this part of Prendergast’s argument builds on an analysis that appears in the classic analysis of Kahn and Huberman (1988) that we discuss in the next section). Thus, up-or-out contracts will be used to increase the incentive for young workers to invest.

Our argument in Proposition 5 is similar to Prendergast’s discussion. In both cases, under standard promotion practices the probability of promotion becomes low when jobs are very similar, and this reduces the effectiveness of standard promotion practices because of a reduction in the incentives for young workers. Further, when this is the case up-or-out becomes attractive because up-or-out increases the probability of promotion and thus increases the incentives for young workers. Where the arguments differ is in terms of the prediction on the “other” dimension, i.e., should up-or-out be associated with a small or large role for firm-specific human capital. As discussed earlier in the section, our approach predicts up-or-out should be employed when firm-specific human capital is unimportant. In contrast, the natural interpretation of Prendergast’s discussion is that up-or-out should be found in settings in which firm-specific human capital is important since in his analysis the role of up-or-out is to increase the investment in firm-specific human capital. As we discuss in the next section, we believe the evidence is more consistent with our approach.

We now summarize our results. Our first main result is that firms will employ up-or-out contracts when firm-specific human capital is low and employ standard promotion practices when it is high. There are a number of reasons for this result including that, as firm-specific human capital becomes small, in the standard-promotion-practices case the misassignment of old workers to jobs due to signaling becomes large while the cost of turnover associated with up-or-out contracts becomes small. Our second main result is that given our basic specification, if jobs are sufficiently similar, then standard promotion practices are preferred if old workers on average are more productive on job 1 while up-or-out is preferred if old workers on average are more productive on job 2. This is the case because, if jobs are sufficiently similar, then both contracts result in no turnover and minimum young-worker effort while standard promotion practices result in all old workers assigned to job 1 while up-or-out contracts result in all being assigned to job 2. Our final main result is that, if firms can commit to a wage floor for promoted
workers, then up-or-out contracts are preferred if jobs are sufficiently similar and young worker effort provision is sufficiently important. The reason here is that in this case up-or-out contracts result in higher young-worker effort.

IV. DISCUSSION

In the previous section we derived conditions in which standard promotion practices are likely to prevail and conditions in which up-or-out contracts are likely to prevail. In this section we discuss two related issues. First, we discuss the extent to which our theoretical results are consistent with real-world observations. Second, we compare and contrast our theoretical approach to up-or-out contracts with other approaches that appear in the literature, where much of our focus is on which theory better matches real-world observations concerning the use of up-or-out contracts.

A) Are the Predictions Correct?

In this subsection we discuss the extent to which our theoretical results are consistent with real-world observations. Since most real-world settings are characterized by standard promotion practices rather than up-or-out contracts, our focus will be on whether the real-world settings characterized by up-or-out contracts are consistent with our theoretical results. In particular, given our belief that in the real world firms have enough commitment ability to commit to wage floors for promoted workers when such commitment significantly improves worker utility, our focus is on whether up-or-out is more likely when there is little firm-specific human capital and when low- and high-level jobs are similar.

We focus on two industries in which up-or-out contracts are common – academia and law. \(^{19}\) Let us begin by discussing academia which we also briefly discussed in the Introduction. The standard employment contract for what are typically referred to as tenure-track (but not tenured) faculty includes a date by which the faculty member must be promoted or the individual is forced to leave. For example, at Cornell the standard for an individual hired straight out of graduate school with his or her degree in hand is that the individual must receive a promotion to associate professor with tenure by the end of the

\(^{19}\) See footnote 5 for a discussion of the potential relevance of our argument to academia. Also, the military employs up-or-out, but we feel the military is a special case and that the use of up-or-out there is likely driven by factors other than the ones we focus on in this paper. See Asch and Warner (2001) for a discussion and analysis of the use of up-or-out in the military.
individual’s sixth year, where if this is not achieved then the individual is given a one-year terminal contract and must leave the university at the end of the terminal contract.\textsuperscript{20}

We believe that academia is characterized by little firm-specific human capital and similar jobs, and thus the fact that up-or-out is common in academia is consistent with our theoretical analysis. Consider first firm-specific human capital. Academia is clearly characterized by some firm-specific human capital, but we believe it is quite limited. For example, one of the authors is a faculty member at the B-school at Cornell and visited for two years at the University of Chicago’s B-school. The courses he taught had to be changed somewhat because Cornell employs a semester system while Chicago uses quarters, but generally the changes required were quite easy to implement. Also, coauthoring with Cornell faculty became somewhat more difficult, but he quickly established a coauthorship with one of the faculty members at Chicago. Or overall, both in terms of that specific transition and we believe more generally, the main activities involved in being a faculty member – research and teaching – are associated with little firm-specific human capital.

Now consider the extent to which in academia the low- and high-level jobs are similar. There are clearly some differences between the job description of a typical assistant professor and that of a typical tenured associate or full professor. For example, a typical associate or full professor would spend more time advising graduate students and mentoring junior faculty than would the typical assistant professor. Similarly, senior faculty tend to spend more of their time on various department and university committees than does the typical untenured assistant professor. But this is mostly an issue of mix of activities or allocation of time than distinct differences in the tasks done by junior and senior faculty. Interestingly, there are distinct differences in the tasks associated with being a senior faculty member and being the chair of a department or a dean. But, consistent with our theory, up-or-out does not apply to those types of promotions.

We now turn our attention to the legal profession. For most of the twentieth century large law firms employed an up-or-out employment system similar to that described above for academia. That is, most large firms hired their associates directly out of law school, where these individuals worked on a probationary basis for a fixed period varying typically between six and ten years. At the end of the period

\textsuperscript{20}In this paper we do not try to provide an explanation for why academia is characterized by tenure. See, for example, Carmichael (1988) for an analysis of this issue.
associates were then considered for promotion to partner, where a rejection meant that the worker was expected to leave the firm within a reasonably short period of time.

The idea that up-or-out was common in large law firms seems consistent to us with our general argument. As discussed by Galanter and Palay (1991) and Kronman (1993), during the period in which up-or-out was dominant corporations tended to employ these firms for routine activities such as commercial contracts, bank loans, and general business advice. In such a setting it is likely that, consistent with our theoretical predictions, firm-specific human capital is limited and low- and high-level jobs are relatively similar. On the one hand, if most of what lawyers did was provide routine legal advice and general business advice, then human capital developed over time should be equally or almost equally valuable at other potential employers as at a worker’s initial employer, i.e., little of the human capital would be firm specific. On the other hand, the provision of routine information and business advice suggests that there is not a clear division of labor between associates and partners, but rather each is doing similar tasks and providing similar services.

Interestingly, since the early 1980s the situation has changed. Many large law firms have moved away from the up-or-out system and now have employment systems with permanent workers that are not partners (see again Galanter and Palay (1991) and also Gorman (1999)). In some firms these workers hold titles such as senior attorney that distinguish them from the probationary workers still called associates, while in other firms these workers are formally referred to as associates while informally they are called permanent associates to distinguish them from their probationary colleagues. Of most interest, however, is that in the typical firm that employs such permanent workers, these positions are staffed both by former probationary associates who have been rejected for partner and by workers hired directly into the positions. In other words, as indicated, these firms have clearly moved away from the up-or-out practice that was dominant in earlier years.

Gorman (1999) attempts to identify the factors that caused this move away from up-or-out in large law firms, where much of her analysis is consistent with the formal theoretical approach taken here. Her approach combines informal theorizing and empirical testing. Her first hypothesis concerns the idea that over time work at large law firms has moved away from the routine legal advice described above towards work which is more complex and knowledge-intensive, where one reason this has occurred is that routine work is now typically handled by in-house legal departments. Her argument is that this increased complexity has resulted in a substantial increase in the firm specificity of human capital, and, consistent
with our formal theoretical model, this increased specificity has caused firms to move away from up-or-out. Her reasoning for why added complexity increases firm specificity is that increased complexity causes workers to develop skills specific to the firm’s particular production processes and to build ties to others in the organization who can provide critical information.

Another of Gorman’s hypotheses concerns the importance of client relationships. In this argument she first points out that partly because routine work is now typically handled by in-house legal departments, client relationships have become less close over the years. That is, corporations are more likely to shop around for quality and price as demands arise, rather than settle on a single law firm for all of their needs. She then argues that, as client relationships have become less important, law firms have moved towards more bureaucratic organizational structures with different departments specializing in various areas of the law such as tax, litigation, and real estate. Then, when a client retains a firm for a specific problem, individuals from the different departments are combined into a temporary team where the partner in charge serves mostly a coordination role. The final step of her argument is that this has caused a move away from up-or-out because, according to earlier work of Pfeffer and Baron (1988), this type of bureaucratic organizational structure favors permanent employment relationships. But note that an alternative interpretation of this last step is that the reorganization of work that has caused partners to take on more of a coordination role has made low- and high-level jobs less similar, and following our theoretical analysis this is what has caused the move away from up-or-out.

After putting forth these arguments, Gorman provides an empirical test based on a cross-sectional analysis of data drawn from the 1996-1997 National Directory of Legal Employers. She finds evidence

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21 Gorman’s third major hypothesis concerns changing social norms, but since this seems unrelated to our theoretical analysis we skip that part of her analysis.

22 One could argue that a weakening of client relationships should result in less firm-specific human capital and that our theory then predicts a decrease in the use of up-or-out. However, we believe this argument is incorrect. Client relationships being weaker means current clients find rival firms to be more feasible as substitutes. In such a world the price charged to a current client will depend on the prices charged by other firms and the abilities of these other firms to serve as effective providers of the demanded legal services. When this is the case, using up-or-out will be disadvantageous because a firm creates more effective competition for its own clients when workers are terminated and move to rivals. Or to put this argument another way, weak client relationships can mean a high level of firm-specific human capital rather than a low level. For example, consider firm A, worker B, and client C, where C is currently A’s client. Further, assume that C prefers A by an amount \( \Delta \) if B is retained, but if B is fired and takes his or her knowledge of C to a rival then C prefers A by an amount \( \Delta/2 \). Then B’s knowledge of C is very valuable to A because retaining the worker allows the firm to charge a premium equal to \( \Delta \) rather than \( \Delta/2 \), but of no value to other firms because C remains with A even if B moves to a rival. In other words, B’s knowledge of C can be regarded as purely firm-specific human capital since it has significant value to A and no value to other firms.
that supports the two hypotheses discussed above. First, up-or-out is less common in firms where the work is more complex. Second, up-or-out is less common in firms where client relationships are less important. Although we do not feel this is strong evidence in favor of our theory since the evidence does not directly concern the importance of firm-specific human capital or the similarity between low- and high-level jobs, we do find the evidence suggestive. That is, since Gorman’s description of the industry connects more complexity to a larger role for firm-specific human capital and weaker client relationships to less similar low- and high-level jobs, we feel that her empirical findings are suggestive of our theoretical approach.

B) Alternative Theories for Up-Or-Out Contracts

In this subsection we compare and contrast our explanation for up-or-out contracts with alternative explanations in the literature. From a theoretical standpoint, the previous argument closest to ours is Kahn and Huberman’s (1988) classic analysis of up-or-out contracts. Kahn and Huberman consider a setting in which workers in their first period at a firm choose whether or not to invest in firm-specific human capital, but directly contracting on the investment is not feasible. They show that under spot contracting underinvestment arises due to a double-moral-hazard problem. That is, ex post firms do not reward old workers who invested when young, and anticipating this workers underinvest. Kahn and Huberman then go on to show that firms can avoid this underinvestment by committing to an up-or-out rule and a high retention wage. By committing in this way firms provide themselves with an incentive to only retain old workers who invested when young, and knowing this workers choose to invest.23

In certain respects our argument is quite similar to Kahn and Huberman’s while in others it is quite different. The papers are similar in terms of the basic role played by up-or-out contracts. In each case standard promotion practices result in an inefficiently small number of promotions which, in turn, can result in inefficiently small incentives for young workers, where this occurs because firms have an incentive to retain old workers but keep them in the low-level/low-paying job. On the other hand, the basic modeling approach is quite different across the two papers. In Kahn and Huberman’s analysis there is no uncertainty concerning worker ability and the incentive problem focused on is the choice of young workers concerning whether or not to invest in firm-specific human capital. In contrast, here there is

23 Waldman (1990) shows that Kahn and Huberman’s argument extends to the case of general-human-capital investment once the promotion-as-signal hypothesis is incorporated into the analysis.
uncertainty and asymmetric learning concerning worker ability and the incentive problem focused on is the effort choice of young workers.

The other main difference between the papers is how the different modeling approaches translate into different predictions concerning when up-or-out is likely to be employed. Related to our discussion of Prendergast (1993) in the previous section, the natural interpretation of Kahn and Huberman’s analysis is that up-or-out should be employed when firm-specific human capital is important since in their analysis up-or-out is used to avoid underinvestment in firm-specific human capital. In contrast, our analysis makes the exact opposite prediction that up-or-out should be more common when firm-specific human capital is of little importance. As discussed above, we believe the empirical evidence is more consistent with our prediction.

Another related analysis appears in Gordanier (2003). Similar to our approach, Gordanier focuses on a tournament explanation for the use of up-or-out contracts, but in contrast to our approach he does not employ the promotion-as-signal hypothesis. Also, another difference between the two papers is that in Gordanier’s model each young worker knows his or her own ability with certainty. In Gordanier’s model up-or-out contracts have the disadvantageous feature that young workers with a low probability of promotion provide no effort, while allowing non-promoted workers to be retained somewhat resolves this low-effort problem. Gordanier further argues that this theory for up-or-out contracts can explain the recent movement away from up-or-out by law firms. Specifically, he argues that the increased emphasis on technical skills has made low effort on the part of young workers more costly to firms, so firms have moved away from up-or-out contracts to lessen the problem of weak incentives. Note that although there are some similarities between Gordanier’s analysis and ours, his paper does not capture the two main predictions we focus on which are that up-or-out is more likely when firm specific human capital is unimportant and jobs are very similar.

A much different approach to up-or-out contracts is found in Demougin and Siow (1994) (see also O’Flaherty and Siow (1995)). In their model, the low-level job, in addition to being used for production, is used both to learn about a worker’s suitability for promotion to the high-level job and train workers for the high-level job. They show that there are two possible equilibria in their setting. In an up-or-out equilibrium, all low-level positions are staffed by young workers and any worker who is not promoted is eventually fired. In what they call a fast-track equilibrium, only some of the low-level
positions are staffed by “trainees” and some workers, possibly all workers, who are never promoted remain with the firm.

In their analysis they derive two predictions concerning when up-or-out is more likely to be employed, i.e., up-or-out is more likely when the level of demand is low and/or growth in demand is high. Low demand generates up-or-out because then worker productivity is low relative to workers’ alternative wages, so the firm wants to minimize the number of workers in the low-level position. The result is that all the low-level positions are used for training and learning. High growth generates up-or-out because in this case there is a large return to training and identifying future managers, so all the low-level positions are staffed by young workers. Turning to Gorman’s (1999) analysis discussed earlier, the evidence concerning these predictions is mixed. On the one hand, Gorman does find that firm growth is related to the use of up-or-out. However, she also finds results that suggest that the level of demand is positively related not negatively related to the use of up-or-out. In particular, she finds that establishments that have a high client business volume employ up-or-out more often not less.

Finally, a number of recent papers have argued that up-or-out is closely tied to the partnership form of organization. For example, Levin and Tadelis (2005) argue that the partnership form of organization arises when firms have greater ability to identify worker ability than clients and the partnership form of organization is then employed to ensure high-quality workers and high-quality output. They go on to argue that up-or-out is employed in combination with partnerships because up-or-out reinforces the tendency of partnerships to only retain the highest ability workers.\(^{24}\) We find the idea of a connection between partnerships and up-or-out to be interesting given that in the world many occurrences of up-or-out are in partnerships rather than corporations. However, we do not think the Levin and Tadelis argument matches well with Gorman’s findings discussed previously. For example, it seems likely that increased complexity would cause worker ability to become more important not less. Thus, the Levin and Tadelis argument suggests a positive correlation between increased complexity and the use of up-or-out but, in fact, as discussed earlier Gorman finds a negative relationship.

\(^{24}\) See also Rebitzer and Taylor (2001) and Morrison and Wilhelm (2004) for arguments that relate the use of up-or-out to the partnership form of organization.
V. CONCLUSION

There is an extensive theoretical literature focused on how the promotion process works in firms characterized by what we refer to as standard promotion processes, while there are also a number of papers that explore the promotion process in firms characterized by up-or-out contracts. Despite this extensive literature, however, only a few papers have considered the factors that determine the choice between standard promotion practices and up-or-out contracts. In this paper we investigate this issue by building a model that combines asymmetric learning and tournament theory similar to the analysis in Zabojnik and Bernhardt (2001). In our model a promotion serves as a positive signal of ability and this results in high wage offers for promoted workers both from prospective employers and from a worker’s current employer. In turn, the possibility of receiving the positive signal and high wage associated with promotion serves as an incentive for young workers to exert effort.

Our analysis yields that there are a number of factors that determine the choice between standard promotion practices and up-or-out contracts. Our main result is that up-or-out contracts are employed when production is characterized by a small amount of firm-specific human capital while standard promotion practices are employed when there is a large amount of firm-specific human capital. We also find that, if firms can commit to a wage floor for promoted workers and effort provision is important, then up-or-out contracts are preferred when low- and high-level jobs are very similar. Further, we feel these results are of interest because they match up well with the choice of standard promotion practices and up-or-out contracts in real-world settings. For example, up-or-out contracts are common in academia where firm-specific human capital is limited and low- and high-level jobs are similar. Also, this perspective helps explain the recent move away from up-or-out contracts in large law firms in that descriptions of recent changes in the nature of work at these firms suggests a growing importance of firm-specific human capital and reduced similarity between low- and high-level jobs.

There are a number of directions in which the analysis in this paper could be extended. Three such directions are as follows. First, in our analysis firms provide young workers with incentives to exert effort, while in the related analyses of Kahn and Huberman (1988), Prendergast (1993), and Zabojnik and Bernhardt (2001) firms provide young workers with incentives to invest in human capital. We have done some preliminary analysis concerning the choice between standard promotion practices and up-or-out in a model closely related to Zabojnik and Bernhardt’s and found results similar to those derived earlier. That is, up-or-out contracts are preferred when firm specific human capital is unimportant and (given firms
have the ability to commit to future wages for promoted workers) when jobs are very similar. We find these preliminary results suggestive, but believe a more complete analysis investigating the robustness of our results to this type of setting is warranted.

Second, in our analysis all firms have access to the same production technology and all workers are ex ante identical. As a result, in our model for any given parameterization it is typically the case that either all firms employ standard promotion practices or all firms employ up-or-out contracts. An interesting extension, therefore, would be to incorporate sufficient firm and worker heterogeneity that equilibrium is characterized by some firms employing standard promotion practices and some employing up-or-out contracts. Extending the analysis in this way might help us better understand how promotion practices vary within industries in real-world settings. Third, in this paper we have assumed that all learning is asymmetric, but our feeling is that the more realistic assumption is that some learning is asymmetric but some is symmetric. That is, some information about a worker’s ability is only directly revealed to the firm employing the worker, while other information is revealed in a public fashion.25 We thus feel it would be worthwhile extending our analysis to see whether additional useful predictions are derived when an element of symmetric learning is introduced.

APPENDIX

Proof of Proposition 1: We first characterize the unique equilibrium in which no one is fired. We start with what happens when a worker is old. Consider wages. Because the initial employer can make counteroffers and beliefs concerning off-the-equilibrium path actions are consistent with each such action being taken by the type with the smallest cost of choosing that action, other firms are willing to offer a worker assigned to job j the worker’s minimum possible output at one of these other firms which is based on who the initial employer assigns to job j in equilibrium. In turn, given the tie-breaking rule assumed, the initial employer just matches these offers and then the worker stays with the initial employer.

Now consider effort levels and job assignments. Since a worker’s wage when old is independent of the worker’s effort level, an old worker will choose the minimum effort level independent of which job the worker is assigned to. Further, since output on job 2 rises faster with effective ability than output on

25 Previous papers that explore symmetric learning include Harris and Holmstrom (1982), Farber and Gibbons (1996), and Gibbons and Waldman (1999b). We are not familiar with any previous papers that mix symmetric and asymmetric learning in the manner we suggest, although related analyses appear in Lazear (1986), Waldman (1990), and Pinkston (2004).
job 1, there must be a value \( \eta_{S'} \) such that old worker \( i \) is assigned to job 1 (job 2) if \( \eta_{it}^e(y_{ijt-1}) < (\geq) \eta_{S'} \) (see footnote 10), where the worker was assigned to job \( j \) in period \( t-1 \). In turn, given the above discussion concerning wages, the wage paid to a worker assigned to job 1 (job 2) is given by
\[
d_1+c_1(\theta_{it}k+e_l) \max \{d_1+c_1(\eta_{S'}^e+e_L),d_2+c_2(\eta_{S'}^e+e_L)\}.
\]

Now consider the value for \( \eta^S \). Suppose \( \theta_{it}k<\eta^S<\theta_{it}k \). Then \( \eta^S \) is the value for \( \eta_{it}^e \) such that a firm is indifferent between assigning an old worker to jobs 1 and 2. In this case \( \eta^S \) satisfies (A1).

(A1) \[
(1+s)[d_1+c_1(\eta^S+e_L)]-(1+s)[d_1+c_1(\theta_{it}k+e_L)]=d_1+c_1(\eta^S+e_L)-d_1+c_1(\theta_{it}k+e_L)
\]
Suppose \( \eta^S=\eta'(e_L,e_L) \). Then (A1) reduces to \( d_1+c_1(\theta_{it}k+e_L)=d_1+c_1(\eta^S+e_L) \), which contradicts \( \eta^S=\eta'(e_L,e_L) \). Suppose \( \eta^S<\eta'(e_L,e_L) \). Then (A1) reduces to (A2).

(A2) \[
(1+s)[d_1+c_2(\eta^S+e_L)]-(1+s)[d_1+c_1(\eta^S+e_L)]=d_1+c_1(\eta^S+e_L)-d_1+c_1(\theta_{it}k+e_L)
\]
But if \( \eta^S<\eta'(e_L,e_L) \), the left-hand side of this expression is strictly negative while the right-hand side is positive so we have a contradiction. Thus, if \( \theta_{it}k<\eta^S<\theta_{it}k \), then \( \eta^S>\eta'(e_L,e_L) \) which, in turn, means
\[
\max \{d_1+c_1(\eta^S+e_L),d_2+c_2(\eta^S+e_L)\}=d_2+c_2(\eta^S+e_L).
\]

Now suppose \( \eta^S=\theta_{it}k \). Consider the return to promoting a worker whose value for \( \eta_{it}^e=\theta_{it}k+\gamma \), \( \gamma \) small. The extra productivity associated with such a promotion equals \( [d_2+c_2(\theta_{it}k+\gamma+e_L)]-[d_1+c_1(\theta_{it}k+\gamma+e_L)] \) which is strictly negative for \( \gamma \) close to zero. We assume that, starting from a situation in which \( \eta^S=\theta_{it}k \), when the off-the-equilibrium path event of a worker not being promoted is observed by the market the inference is that the worker’s expected effective ability is \( \theta_{it}k \) (this follows from our assumption concerning off-the-equilibrium path actions). The extra cost of promoting such a worker is therefore zero. Thus, since the extra productivity of promoting such a worker is less than the extra cost, the firm will not want to promote the worker so we have a contradiction. Hence, \( \eta^S>\eta'(e_L,e_L) \).

Now consider young-worker job assignments and wages. Given that from above we know that a firm earns positive expected profits from an old worker who it employed when young, competition across firms means that the wage for young workers must exceed expected productivity. We also know that, given our assumption \( d_1+c_1(\theta_{it}e_H)>d_2+c_2(\theta_{it}+e_H) \) and that in our specification the rate of learning is independent of job assignment, all young workers are assigned to job 1.

Now consider young-worker effort. Given our assumption that \( g(e) \) is sufficiently convex (see footnote 6), all young workers must choose the same effort level that we denote \( e_Y^S \). Combining this result with previous results yields \( w_Y^S>d_1+c_1(\theta^*+e_Y^S) \). Also, as shown in the proof of Lemma 1 below, for \( s \) sufficiently small young-worker effort equals the minimum value \( e_L \) so clearly \( e_Y^S \) can be less than \( e_Y^* \). More generally, changes in \( s \) affect \( e_Y^S \) but do not affect \( e_Y^* \) with the result that \( e_Y^S \) can be greater than, less than, or equal to \( e_Y^* \).

We now argue that, if \( s \) is sufficiently large, then there does not exist an equilibrium in which workers are fired. Suppose some workers are fired and the lowest ability worker retained is assigned to job 1. Using the same logic as above, if some workers are fired then there must exist a critical value, call it \( \eta^S \), such that workers are fired if \( \eta_{it}^e(y_{ijt-1})<\eta^S \) and retained if \( \eta_{it}^e(y_{ijt-1})>\eta^S \). Suppose \( \eta^S<\eta'(e_L,e_L) \).
Then the wage for an $\eta^S$ worker who is retained equals $d_1+c_1(\eta^S+e_L)$. But given this wage, a firm could keep a worker with expected effective ability equal to $\eta^S-\gamma$, $\gamma$ small, and earn strictly positive profits which is a contradiction. Suppose $\eta^S>\eta'(e_L,e_L)$. Then the wage for an $\eta^S$ worker who is retained equals $d_2+c_2(\eta^S+e_L)$. Suppose $s$ is such that $(1+s)(d_1+c_1(\theta_hk+e_L))>[d_2+c_2(\theta_hk+e_L)]$. Given this condition, a firm could keep a worker with expected effective ability equal to $\eta^S-\gamma$, $\gamma$ small, and earn strictly positive profits which is a contradiction. Hence, if $s$ is sufficiently large, there cannot be an equilibrium in which some workers are fired and the lowest ability worker retained is assigned to job 1. Finally, a similar argument yields that, if $s$ is sufficiently large, there also cannot be an equilibrium in which some workers are fired and the lowest ability worker retained is assigned to job 2.

Proof of Proposition 2: We start with what happens when a worker is old. Consider first wages. Because the initial employer can make counteroffers and because beliefs concerning off-the-equilibrium path actions are consistent with each such action being taken by the type with the smallest cost of choosing that action, other firms are willing to offer a retained old worker the worker’s minimum possible output at one of these other firms which is based on who the initial employer retains in equilibrium. In turn, given the tie-breaking rule assumed, the initial employer just matches these offers and then the worker stays with the initial employer. For workers who are not retained, competition across firms means such a worker is paid expected output at one of these other employers.

Now consider effort levels and who is retained. Since a worker’s wage when old is independent of the worker’s effort level, an old worker will choose the minimum effort level independent of which firm the worker is employed at and which job the worker is assigned to. Further, since output on job 2 rises with effective ability, there must be a value $\eta^{U+}$ such that old worker $i$ is retained (not retained) if $\eta_i^{e}(y_{ijt-1})\geq(<)\eta^{U+}$, where the worker was assigned to job $j$ in period $t-1$. In turn, given the above discussion concerning wages, the wage paid to a retained old worker is given by $\max\{d_1+c_1(\eta^{U+}+e_L),d_2+c_2(\eta^{U+}+e_L)\}$ while the wage paid to a fired worker equals $\max\{d_1+c_1(E(\theta)k+e_L),d_2+c_2(E(\theta)k+e_L)\}$.

Now consider the value for $\eta^{U+}$. If $\theta_hk<\eta^{U+}<\theta_hk$, then this is the value for $\eta_i^{e}$ such that a firm is indifferent between retaining and firing an old worker. Thus, in this case $\eta^{U+}$ satisfies (A3).

\begin{align*}
(1+s)[d_2+c_2(\eta^{U+}+e_L)]-\max\{d_1+c_1(\eta^{U+}+e_L),d_2+c_2(\eta^{U+}+e_L)\}=0
\end{align*}

Suppose $\eta^{U+}=\eta'(e_L,e_L)$. Then (A3) reduces to $s[d_2+c_2(\eta^{U+}+e_L)]=0$ which is a contradiction. Thus, if $\theta_hk<\eta^{U+}<\theta_hk$, then $\eta^{U+}=\eta'(e_L,e_L)$ which, in turn, means $\max\{d_1+c_1(\eta^{U+}+e_L),d_2+c_2(\eta^{U+}+e_L)\}=d_1+c_1(\eta^{U+}+e_L)$.

Now suppose $\eta^{U+}=\theta_hk$. Consider the return to retaining a worker whose value for $\eta_i^{e}=\theta_hk-\gamma$, $\gamma$ small. The productivity associated with such a retention equals $(1+s)[d_2+c_2(\theta_hk-\gamma+e_L)]$. We assume that, starting from a situation in which $\eta^{U+}=\theta_hk$, when the off-the-equilibrium path event of a worker being retained is observed by the market the inference is that the worker’s expected effective ability is $\theta_hk$ (this follows from our assumption concerning off-the-equilibrium path actions). Thus, the cost of retaining the
worker is \([d_2+c_2(\theta_i k+e_L)]\). For \(\gamma\) sufficiently small the productivity associated with retaining such a worker exceeds the cost of retaining the worker, so the firm will want to retain the worker which is a contradiction. Hence, \(\eta^{U+}<\eta^c(e_L,e_L)\) and the wage paid to a retained old worker equals \(d_1+c_1(\eta^{U+}+e_L)\).

Now consider young-worker job assignments and wages. Given that from above we know that a firm earns positive expected profits from an old worker who it employed when young, competition across firms means that the wage for young workers must exceed expected productivity. We also know that, given our assumption \(d_1+c_1(\theta_i k+e_L)>d_2+c_2(\theta_i k+e_L)\) and that in our specification the rate of learning is independent of job assignment, all young workers are assigned to job 1.

Now consider young-worker effort. Given our assumption that \(g(e)\) is sufficiently convex (see footnote 6), all young workers must choose the same effort level that we denote \(e_{Y^{U}}\). Combining this result with previous results yields \(w_{Y}\) \(>d_1+c_1(\eta^{U+}+e_L)\). Also, as shown in the proof of Lemma 1 below, for \(s\) sufficiently large young-worker effort equals the minimum value \(e_L\) so clearly \(e_{Y^{U}}\) can be less than \(e_Y^*\).

More generally, changes in \(s\) affect \(e_{Y^{U}}\) but do not affect \(e_Y^*\) with the result that \(e_{Y^{U}}\) can be greater than, less than, or equal to \(e_Y^*\).

Proof of Lemma 1: Given our finding that \(\eta^{S^+}>\eta^c(e_L,e_L)\), (A1) can be rewritten as (A4).

\[(1+s)[d_1+c_1(\eta^{S^+}+e_L)]-[d_1+c_1(\theta_i k+e_L)]=(1+s)[d_2+c_2(\eta^{S^+}+e_L)]-[d_2+c_2(\eta^{S^+}+e_L)]\]

In turn, (A4) can be rewritten as (A5).

\[s[d_1+c_1(\theta_i k+e_L)]+(1+s)c_1(\eta^{S^+}-\theta_i k)=s[d_2+c_2(\theta_i k+e_L)]+sc_2(\eta^{S^+}-\theta_i k)\]

Suppose \(sc_2 \leq (1+s)c_1\). Given \(d_1+c_1(\theta_i k+e_L)>d_2+c_2(\theta_i k+e_L)\), we now have a contradiction. Thus, if \(sc_2 \leq (1+s)c_1\), then \(\eta^{S^+}=\theta_i k\). But this means there must exist a largest value for \(s\), call it \(s^{S^+}\), such that \(\eta^{S^+}=\theta_i k\) for all \(s \leq s^{S^+}\).

Now consider values \(s_1\) and \(s_2\), \(s_1>s_2>s^{S^+}\). Also, let \(\eta^{S^+}(s)\) be the function that gives equilibrium values for \(\eta^{S^+}\) as a function of \(s\). Given \(\eta^{S^+}(s_2)<\theta_i k\) and \(d_2+c_2(\eta^{S^+}(s_2)+e_L)>d_1+c_1(\eta^{S^+}(s_2)+e_L)\), (A1) yields (A6).

\[(1+s_2)[d_1+c_1(\eta^{S^+}(s_2)+e_L)]-[d_1+c_1(\theta_i k+e_L)]=(1+s_2)[d_2+c_2(\eta^{S^+}+e_L)]-[d_2+c_2(\eta^{S^+}+e_L)]\]

In turn, (A6) yields (A7).

\[s_2[d_1+c_1(\theta_i k+e_L)]+(1+s_2)(\eta^{S^+}(s_2)-\theta_i k)=s_2[d_2+c_2(\theta_i k+e_L)]+s_2c_2(\eta^{S^+}(s_2)-\theta_i k)\]

Dividing by \(s_2\) and rearranging terms yields (A8).

\[(\eta^{S^+}(s_2)-\theta_i k)[(c_1/s_2)-(c_2-c_1)]=[d_2+c_2(\theta_i k+e_L)]-[d_1+c_1(\theta_i k+e_L)]\]

Since the right-hand side of (A7) is negative, we have that \((c_1/s_2)-(c_2-c_1)<0\). Suppose now \(s\) increases from \(s_2\) to \(s_1\). The right-hand side of (A7) is unchanged while \((c_1/s_2)-(c_2-c_1)<(c_1/s_2)-(c_2-c_1)\). For (A8) to still hold we now have that \(\eta^{S^+}(s_1)-\theta_i k<\eta^{S^+}(s_2)-\theta_i k\) or \(\eta^{S^+}(s_1)<\eta^{S^+}(s_2)\).

Given our finding that \(\eta^{U+}<\eta^c(e_L,e_L)\), (A3) can be rewritten as (A9).

\[(1+s)[d_2+c_2(\eta^{U+}+e_L)]-[d_1+c_1(\eta^{U+}+e_L)]=0\]
Consider the value for s, call it $s^\#$, such that $(1+s^\#)(d_2+c_2(\theta_1 k+e_L))-d_1+c_1(\theta_1 k+e_L)=0$. Then clearly (A9) will not be satisfied with $\eta^{U^+} > \theta_1 k$ for any $s \geq s^\#$. But this means there must exist a smallest value for s, call it $s^{U^+}$, such that $\eta^{U^+} = \theta_1 k$ for all $s \geq s^{U^+}$.

Now consider values $s_1$ and $s_2$, $s_1 < s_2 < s^{U^+}$. Also, let $\eta^{U^+}(s)$ be the function that gives equilibrium values for $\eta^{U^+}$ as a function of s. Given $\eta^{U^+}(s_1) > \theta_1 k$ and $\eta^{U^+}(s_1) < \eta'(c_1, e_L)$, (A3) yields (A10).

(A10) $$(1+s_1)(d_2+c_2(\eta^{U^+}(s_1)+e_L))-d_1+c_1(\theta_1 k+e_L)=0$$

In turn, (A10) yields (A11).

(A11) $$s_1[d_2+c_2(\eta^{U^+}(s_1)+e_L)]+(c_2-c_1)(\eta^{U^+}(s_1)-\theta_1 k)=[d_1+c_1(\theta_1 k+e_L)]-d_2+c_2(\theta_1 k+e_L)]$$

Suppose now s increases from $s_1$ to $s_2$. Since the right-hand side of (A11) is unchanged, for (A11) to still hold we now have that $\eta^{U^+}(s_2) < \eta^{U^+}(s_1)$.

Now consider effort levels. Consider first the case of standard promotion practices where $s \leq s^{S^+}$. We know no one is promoted, i.e., $\eta^{S^+} = \theta_1 k$. But given the noise term in equation (2) has a normal distribution, this means a worker’s wage when old is independent of his or her output when young so there is no return for a young worker increasing his or her effort level. Hence, since effort is costly, young workers must choose $e_Y^{S^+} = c_L$. Similarly, consider the case of up-or-out contracts where $s \geq s^{U^+}$. We know that everyone is retained, i.e., $\eta^{U^+} = \theta_1 k$. But again given the noise term in equation (2) has a normal distribution, this means a worker’s wage when old is independent of his or her output when young so there is no return for a young worker increasing his or her effort level. Hence, since effort is costly, young workers must choose $e_Y^{U^+} = c_L$.

Equation (A8) tells us that $\eta^{S^+}(s)$ is a continuous function of s, so $\eta^{S^+}(s)$ must approach $\theta_1 k$ as s approaches $s^{S^+}$ from above. Further, we know that a young worker always has the option of choosing the minimum effort level and doing no worse than be assigned to job 1 with probability one when he or she becomes old. Given this, as s approaches $s^{S^+}$ from above and $\eta^{S^+}(s)$ approaches $\theta_1 k$, i.e., as the probability of promotion approaches zero, the idea that equilibrium effort must do at least as well as choosing the minimum effort yields $e_Y^{S^+}$ must approach $e_L$.

Equation (A11) tells us that $\eta^{U^+}(s)$ is a continuous function of s, so $\eta^{U^+}(s)$ must approach $\theta_1 k$ as s approaches $s^{U^+}$ from below. Given this, consider what happens as s approaches $s^{U^+}$ from below. Given the noise term in equation (2) is normal and given $\eta^{U^+}(s)$ approaches $\theta_1 k$ as s approaches $s^{U^+}$ from below, i.e., the probability of retention approaches one, the probability of retention must also approach one given a worker chooses the minimum effort level. Combining this with the idea that equilibrium effort must do at least as well as choosing the minimum effort level yields that, as s approaches $s^{U^+}$ from below and $\eta^{U^+}(s)$ approaches $\theta_1 k$, the equilibrium effort level $e_Y^{U^+}$ must approach $e_L$.

Proof of Proposition 3: Consider a value for s that satisfies $s < \min\{s^{S^+}, s^{U^+}\}$. Given Lemma 1 we know that if a young worker chooses a firm that employs standard promotion practices, then $\eta^{S^+} = \theta_1 k$ and $e_Y^{S^+} = 0$. On the other hand, from Lemma 1 we also know that if a young worker chooses a firm that employs up-or-
out contracts, then \( \theta_L k < \eta^U < \theta_H k \) which, in turn, means \( e_Y^U > 0 \) since workers will want to expend more than the minimum effort level because increasing effort improves beliefs about expected effective ability and thus the probability of receiving the higher wage associated with retention in the following period. It is also the case that, as \( s \) approaches zero, the efficiency loss due to some old workers switching firms goes to zero and the efficiency loss concerning the misassignment of old workers to jobs goes to zero, i.e., \( \eta^U \) approaches \( \eta'(e_L, e_L) \) (see equation (A10)).

Because competition across firms means all firms earn zero profits in equilibrium, workers receive any surplus in equilibrium. This means workers will choose the contract that maximizes efficiency, or equivalently, social surplus. From above we know that, as \( s \) approaches zero, up-or-out contracts have a more efficient assignment of old workers to jobs, the inefficiency loss due to the misassignment of old workers to firms goes to zero, and a more efficient young-worker effort level. The last statement must be true since \( e_Y^S = e_L \) if \( s \) is sufficiently small, \( e_Y^U > e_L \) if \( s \) is sufficiently small, and our assumption that \( c_1 e_1 - \alpha g(e_1) > c_1 e_L - \alpha g(e_L) \) which implies that increasing effort above \( e_L \) must be efficiency enhancing. Thus, up-or-out contracts will be preferred if \( s \) is sufficiently small.

Now consider a value for \( s \) that satisfies \( s > \max\{s^S, s^U\} \). Given Lemma 1 we know that if a young worker chooses a firm that employs up-or-out contracts, then \( \eta^U = \theta_L k \) and \( e_Y^U = 0 \). On the other hand, from Lemma 1 we also know that if a young worker chooses a firm that employs standard promotion practices, then \( \theta_L k < \eta^S < \theta_H k \) which, in turn, means \( e_Y^L > 0 \) since workers will want to expend more than the minimum effort level because increasing effort improves beliefs about expected effective ability and thus the probability of receiving the higher wage associated with promotion in the following period. It is also the case that, as \( s \) approaches \( \infty \), the assignment of old workers to jobs is superior for standard promotion practices, i.e., \( \eta^U = \theta_L f(1) \) while \( \eta^S \) approaches \( \eta'(e_L, e_L) \).

As before, because workers receive any surplus in equilibrium, workers choose the contract that maximizes social surplus. From above, we know that as \( s \) approaches \( \infty \) standard promotion practices have a more efficient assignment of old workers to both firms and jobs and a more efficient young-worker effort level (the latter follows for reasons similar to an argument above concerning what happens when \( s \) is small). Hence, standard promotion practices will be preferred if \( s \) is sufficiently large.

**Proof of Lemma 2:** From the proof of Lemma 1 we know that, if \( sc_2 \leq (1+s)c_1 \), then \( \eta^S = \theta_H k \). Hence, a normalized increase in \( c_1 \) that makes \( c_2 - c_1 \) sufficiently small, i.e., makes \( sc_2 \leq (1+s)c_1 \), will result in \( \eta^S = \theta_H k \) and from earlier we know this means \( e_Y^S = e_L \). Now consider up-or-out contracts. In the limit, as \( c_2 - c_1 \) goes to zero a normalized increase in \( c_1 \) causes the difference \( [d_2 + c_2 (\theta_H k + e_L)] - [d_1 + c_1 (\theta_L k + e_L)] \) to approach zero. Given this, (A9) tells us that a normalized increase in \( c_1 \) that causes \( c_2 - c_1 \) to become sufficiently small will result in \( \eta^U = \theta_L k \) and \( e_Y^U = e_L \).
Proof of Proposition 4: As argued earlier, workers will choose the contract that maximizes social surplus. From Lemma 2 we know that a normalized increase in $c_1$ that makes $c_2-c_1$ sufficiently small results in $\eta^S = \theta H_k$, $e_Y^S = e_L$, $\eta^U = \theta L_k$, and $e_Y^U = e_L$. In other words, the only difference from an efficiency standpoint between the two contracts is that all old workers are assigned to job 1 under standard promotion practices while all old workers are assigned to job 2 under up-or-out contracts. But this means workers choose standard promotion practices when an average ability old worker is more productive on job 1, i.e.,

$$(1+s)[d_1+c_1(0'k+e_L)] > (1+s)[d_2+c_2(0'k+e_L)]$$

while workers choose up-or-out contracts when an average ability old worker is more productive on job 2, i.e.,

$$(1+s)[d_1+c_1(0'k+e_L)] < (1+s)[d_2+c_2(0'k+e_L)]$$

Further, when an average ability old worker is equally productive at the two jobs, i.e.,

$$(1+s)[d_1+c_1(0'k+e_L)] = (1+s)[d_2+c_2(0'k+e_L)]$$

both types of contract are consistent with equilibrium behavior.

Proof of Proposition 5: Consider first the case of standard promotion practices. As shown in the proof of Lemma 2, as $c_2-c_1$ gets sufficiently small, in the absence of wage floors the high wage associated with a promotion due to the signal stops any promotions from occurring. Imposing a wage floor for old workers assigned to job 2 does not affect this result. Hence, now that such a wage floor is possible it is still the case that $\eta^S = \theta H_k$ and $e_Y^S = e_L$.

Now consider up-or-out contracts. As shown in the proof of Lemma 2, as $c_2-c_1$ gets sufficiently small, in the absence of wage floors $\eta^U = \theta L_k$ and $e_Y^U = e_L$. Further, given the counteroffer assumption, the wage associated with being retained equals $d_1+c_1(0_hk+e_L)$ while the expected productivity associated with retaining an old worker whose expected effective ability is $\theta_L$ equals $(1+s)[d_2+c_2(0_Lk+e_L)]$ which is necessarily greater than $d_1+c_1(0_hk+e_L)$ for $c_2-c_1$ sufficiently small. Suppose the firm commits to a wage floor for retained old workers equal to $(1+s)[d_2+c_2(0_Lk+e_L)]+\gamma$, $\gamma>0$ but small. Then an old worker will be retained with probability less than one which, in turn, means that young-worker effort will be above the minimum value.

Now consider the value for $e_Y^U$ in more detail. $e_Y^U$ will be a decreasing function of $\alpha$ where, if unconstrained, $e_Y^U$ would grow without bound for $\alpha$ sufficiently close to zero. Given the upper bound on effort choice, we have that for $\alpha$ sufficiently small $e_Y^U$ equals this upper bound and also equals the first-best choice of $e_Y$ (this follows from the unconstrained value for the first best which is defined by $\alpha g'(e)=c_1$ going to $\infty$ as $\alpha$ goes to zero). Further, if this upper bound is sufficiently large as is assumed in footnote 18, then the higher effort associated with up-or-out contracts means that these contracts will be preferred for $\alpha$ sufficiently small.

REFERENCES


