Subsidies and capital markets: implications for microfinance loan portfolios

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Abstract

We model a microfinance lender that receives subsidized funding from external investors who value the social impact of the lender. The impact depends on the lender's type, which is not observable to the investors. In a pooling equilibrium, the subsidy raises the lender's profit but distorts the loan portfolio choice of the low quality lender. The lender's portfolio choice can be improved two different ways. One is through a separating equilibrium, where the lender specializes in the type of lending at which he is most effective. The other is through arms-length contracting, characterized by less informed external investors. In this case, less information implies that the lender wastes less resources trying to justify access to subsidies.

JEL Codes: G21; O16; D86

1. Introduction

A persistent problem for many developing economies is the difficulty small and micro-sized businesses have in accessing credit. In bridging this gap, one success has been microfinance. Facing numerous obstacles, microfinance has proved capable of connecting entrepreneurs with exceptionally small business loans. As microfinance has matured over the years, subsidies remain important to the industry. In a survey of institutions located in 49 countries, Cull et al. (2007) find that on average 20% of an institution's funding is in the form of a subsidy. The exact role of subsidies in microfinance is a source of debate. While many analysts agree that subsidies have a place in microfinance, there is growing concern about how continued subsidization is influencing the behavior and development of these institutions.[1]

To examine this question, we develop a two-period model with a single microfinance lender. In the first period, the lender issues small loans to the poor. These loans generate gains for the borrowers, but the size of the gains depends on the quality of the lending program. This is important because the lender's payoff is derived in part from these gains. That is, consumption by the poor generates a positive externality, which we call social benefit. In the second period, the lender learns two things. The lender learns the size of the benefits that are generated from his small lending program and the lender learns about new lending opportunities. These opportunities take the form of larger business loans, which, while more profitable, do not generate any social benefit.

The microfinance lender acts as an intermediary, transforming external funding into loans. The external funding is supplied by social investors, in the form of debt.[2] The supply of funds depends on two important aspects of the institution's lending program. One is the degree of risk in the loan portfolio, which the lender can control by undertaking prudent, but costly measures. To ensure prudent lending, the investors demand that the microfinance lender finance part of each loan with a

[1] See Morduch (2005) for a discussion of these issues.

[2] See Conning and Morduch (2011) for a discussion of the role of social investment in microfinance.

sufficient amount of equity capital.[3] Second, social investors are willing to subsidize the supply of external funds because like the microfinance institution, they benefit from the positive externality. However, unlike the microfinance institution, the social investors are unable to observe the exact size of the benefit created by the institution's small lending program.

In this setting, the microfinance lender can face a dilemma. On the one hand, the lender can choose to ignore funding implications and simply issue the type of loans that maximize return on capital. The other option is that the lender can distort his loan portfolio in order to preserve access to external subsidies. This decision is driven by both the extent of the subsidy and the profitability of the larger loan opportunities. One possible outcome is a pooling equilibrium, where different types of microfinance lending programs all use subsidized funding to issue small loans to the poor. This behavior has two implications. One is that subsidized resources end up being used to fund small loan programs that have little or no social impact, and the other is that the microfinance lender refuses to cater to his more successful clients who demand larger loans sizes.

Aside from this, we show that subsidized external funding makes microfinance lending more profitable, which encourages financial intermediation. Also, we find that subsidies need not necessarily distort portfolio choice. In another equilibrium outcome, there is an efficient form of separation, where the two types of microfinance lender self-select according to the type of lending that they are good at. In this case, only microfinance lenders who are effective at poverty reduction receive subsidies, while the other type pays the market rate for the debt funding.

The results of our study contribute to a large theoretical literature on microfinance and contracting. Hoff and Stiglitz (1998) and Bose (1998) have models that examine how policy intervention in the formal banking sector can have unintended consequences for informal lenders. McIntosh and Wydick

[3] In our model, the capital requirement is an endogenous result of market discipline. In practice, such requirements are often part of prudential regulation aimed at protecting depositors. See Pouchous (2012) for an overview of microfinance regulation. Also, keep in mind that capital requirements can be enforced for non-deposit taking microfinance institutions as well, such as in Ghana and Peru. See Steel and Andah (2003) and Ebentreich (2005).

(2005) consider how a socially minded microfinance lender may want to cross-subsidize across different kinds of loans in his portfolio, depending on the competitive pressure exerted by outside lenders. Mansuri (2007) models a credit market in a developing country where an upstream source of funds finds it profitable to delegate loan provision to downstream lenders, who hold advantages in information and enforcement. In a similar fashion, Jain (1999), Varghese (2005) and Andersen and Malchow-Moller (2006) examine low income credit markets where borrowers can solicit loans from different types of lenders, such as money lenders or banks, who in turn, hold different kinds of advantages.

Our own model assumes that the lending process involves two layers, like Mansuri (2007). In particular, we study how upstream investors contract with a downstream microfinance lender that is heterogenous in type. One of our main contributions is to illuminate how the contracting between the microfinance lender and external investors influences lending behavior and poverty reduction. Between 2005 and 2010 foreign investment in microfinance climbed from roughly $2 billion to $13 billion, according to Reille et al. (2011). With this increase in funds going to microfinance, the composition of funding has been changing. In the early years, microfinance tended to be funded by a single donor or development organization, who closely monitored the institution. More recently, microfinance has gained access to more diverse sources of funds, through securitization and access to international capital markets. A good part of this funding comes from social investors, who seek both financial returns and social outcomes. A potential trade-off is that this new class of investor is often described as more arms-length in nature.[4]

To model this evolution of funding, we consider two types of contracting between the microfinance institution and the external investors. First, we look at a case where the external investors can contract directly on the type of loans that the institution issues and second, we look at an arms-length contract where they cannot. Interestingly, we find that the latter contract has several advantages. For one, it

[4] While diverse, international investors may be relatively less informed, Bugg-Levine et al. (2012) point out that the growing field of "impact investing" is making progress towards a more objective and clear measurment of social outcomes.

makes microfinance intermediation more profitable. While this may lead to a misuse of subsidies, we find that the negative implications are minimized through the endogenous behavior of the institutions. Also, we find that in a dynamic framework, the leniency in the debt contracts directly impacts the capital requirement. By raising the discounted expected payoff from intermediation, the first period capital requirement falls, which allows the institution to issue more loans and potentially achieve more poverty reduction. This result has some similarity with Hellman et al. (2000) and Repullo (2004), who find that higher capital requirements lead to a lower franchise value for the intermediary, which can have unintended implications, such as riskier lending behavior.

A small, but growing collection of empirical papers have begun to examine how external funding is related to the individual microfinance institution. For example, Garmaise and Natividad (2010 & 2013) identify different factors that appear to influence what a microfinance lender pays for his external funds. There are also papers like Caudill et al. (2009), Hermes et al. (2011), Hudon and Traca (2011) and Bogan (2012) that investigate whether external subsidies are associated with inefficiencies within individual microfinance institutions. In addition, D'Espallier et al. (2013) compare microfinance lending behaviors and find that institutions that are not subsidized adopt different strategies than their subsidized counterparts, such as lending to richer clients and reducing the share of female borrowers.

A key part of our model is a choice the microfinance lender faces in the second period regarding the size of loans. Small loans offer an advantage in that they can create social benefit, which is enjoyed by both the microfinance lender and external investors. However, in practice not all microfinance lending programs turn out to deliver these benefits. In a review of the empirical literature, Bauchet et al. (2011) point out that a number of randomized control studies find that the treatment effects of various programs is low. Interestingly though, while average treatment effects of microcredit are small, several studies find that a subset of the clients do indeed benefit, often significantly. In their survey of the empirical work, Bauchet et al. (2011) emphasize this effect on the tail and argue that an important agenda is how to better identify ex ante, this high ability type of microfinance borrower.

The idea that microfinance institutions are in a unique position to exploit profitable lending opportunities is supported by the literature on relationship banking. In relationship lending, lending opportunities arise from the soft information that is accumulated by loan officers.[5] To understand whether lenders choose to pursue these opportunities or not, the literature tends to focus on the degree of external competition, as in Petersen and Rajan (1995) and Boot and Thakor (2000), or the size and/or organizational structure of the loan issuing bank, as in Berger and Udell (2002) and Mian (2006). Our approach explores how the supply of external funding can explain the loan portfolio choices of a bank.

We have organized our paper in the following manner. In Section 2 we describe the model and introduce a two-period game between a microfinance institution and external investors. Section 3 is dedicated to examining equilibrium behavior. To organize the analysis, we break the discussion into sub-sections. In Section 3.1 we study the portfolio and fund raising choices of the microfinance lender in the second period of the game. In Section 3.2, to establish a benchmark, we examine a case where the microfinance lender does not receive subsidies. In Section 3.3 we then examine equilibrium behavior when external investors provide subsidized funds under a contract where the investors can monitor the portfolio choice of the microfinance lender. In this case, we describe two different equilibrium outcomes. In Section 3.4 we introduce arms-length funding, by assuming that investors are unable to monitor portfolio choice. In this case, we identify a single pooling equilibrium. Next, in Section 3.5 we explore the dynamic implications of the different kinds of contracting. Finally, in Section 4 we have the conclusion.

2. The model

2.1 The microfinance institution and the external investors

Consider a two-period model with a single microfinance institution (MFI). The MFI acts as an intermediary, transforming external funding into one-period loans. The MFI has access to two kinds

[5] For example, see Sharpe (1990), Petersen and Rajan (1994) and Canales and Nanda (2012).

of funding. One type is equity funding. The MFI raises equity in the amount of X at the start of each period, where X is a parameter and has a cost ρ. The other type of funding is external debt, which is supplied by social investors. Social investors have unlimited funds at an opportunity cost of $i_m > 0$. We refer to i_m as the market interest rate and assume that $\rho > i_m$.

The MFI starts each period by choosing how much external debt to raise from social investors. In period one, the MFI then uses his combined funding (capital plus debt) to issue \$1 loans. Each loan stipulates a loan repayment of $1 + r$, where the interest rate $r > 0$ is a parameter.[6] When the MFI issues the loans, the MFI has a choice whether to adopt prudent lending practices or not. If the MFI lends prudently, then the MFI faces an operating cost of c per loan and the loan is repaid with certainty. If the lender chooses to not lend prudently, then operating cost is 0 and the loan is repaid with probability λ, where $\lambda \in (0, 1)$. We assume that under imprudent lending, the loan repayments are perfectly correlated.

Assumption A1 $\quad r - c - i_m > 0$

This assumption implies that it is efficient to fund prudent lending.

In period two, after raising external debt, the MFI again uses his combined funds to issue loans. However, in period two, the MFI is given a choice over the size of the loans. The MFI can either issue \$1 loans as before, or alternatively, issue loans of size \$$b$, where $b > 1$ and b is a parameter. On a b loan, the loan repayment is $(1 + r)b$. In addition to loan size, the MFI must also choose whether to use prudent lending practices or not, as in period one. If the MFI lends prudently, then the operating cost is c, regardless of loan size. This implies that the cost of the loan, per dollar, is inversely related to loan size.[7]

[6] We choose to treat r as a parameter in order to maintain focus on the relationship between the MFI and external investors. Nonetheless, the assumption is not exactly harmless, as the interest rate is certainly related to social impact. As a justication, one might consider r in our paper to be the "going rate" in the credit market.

[7] A number of empirical studies document relatively high operating costs per dollar in microfinance. For example,

Assumption A2 $\quad (1-\lambda)(1+r) > c > (1-\lambda)rb$

Assumption A2 governs the moral hazard problem involving the MFI's decision about prudent lending. The advantage of not being prudent is that it saves the MFI the transaction cost, c. The disadvantage is that it exposes the portfolio to a risk of default, namely $1-\lambda$. As we will confirm later, the incentive to be prudent rises as the MFI uses more capital and less debt. Under A2, the MFI prefers prudent lending as long as a minimum fraction of the loan is financed using capital.

Both the MFI and the social investors are assumed to be risk neutral. The payoffs to the MFI and the social investors are a linear combination of what we call the financial payoff and social payoff. The MFI's one-period financial payoff is defined as the loan repayments from his borrowers, net of transaction costs and external funding obligations. The social investors' financial payoff consists of the principal and interest repaid by the MFI, net of the cost, $1 + i_m$.

The social payoffs work as follows. While we do not model borrower behavior in our paper, presumably the borrowers realize gains from consumption. Assuming these borrowers have incomes below the poverty line, we say that this consumption resulting from microfinance loans generates a positive externality. More specifically, for every \$1 loan that is repaid to the MFI, the MFI receives a social benefit of Δ_j, measured in dollars, where j indicates the type of MFI that is issuing the loan. On the same \$1 loan, the social investors also receive a social benefit of Δ_j, per dollar of external debt supplied to the MFI.[8] In contrast, the big loans are assumed to go to the non-poor, and hence, this

in their survey Cull et al. (2009) find that the median NGO has a loan size that is less than a quarter the size of the average loan provided by a commerical bank that offers microloans, and correspondingly, operating costs as a fraction of loan value are roughly double that of the bank.

[8] There is some double counting here. Both the MFI and investors earn social benefits from the same \$1 loan. This is because the social benefits have non-rivalry properties. Also, note that the MFI receives Δ_j on each \$1 loan, regardless of how the loan is funded. In contrast, the social benefit going to the investors is only Δ_j per dollar of debt supplied. For example, say an MFI funds ten \$1 loans using \$3 in capital and \$7 in debt. Then the investors receive a social payoff of $7\Delta_j$.

kind of consumption does not generate social benefit.[9]

The social benefit that the MFI and social investors derive from a small loan depends on the type of MFI that issues the loan. In a survey of empirical studies, Bauchet et al. (2011) point out that the variation in social impact realized under different lending programs is often correlated with specific features of the program, such as the borrower's gender, whether the borrower has an existing businesses, the timing of loan repayments, group liability, and others. To capture this, we assume that there are two types of MFI in the model, denoted as h and l, and that $\Delta_h > \Delta_l$. Also, in order to simplify the analysis, we assume that $\Delta_l = 0$.

Assumption A3 $\quad 1 > \lambda(1 + r + \Delta_h)$

Assumption A4 $\quad i_m \geq \Delta_h$

If the MFI chooses imprudent lending, then the probability of receiving the loan repayment and earning social benefits is only λ, as opposed to 1. Assumption A3 implies that this choice is sufficiently risky to make funding the loan undesirable from the point of view of the external investors. With regard to Assumption A4, note that the social investors view the interest rate and social impact as perfect substitutes. Hence, if $\Delta_h > i_m$, then social investors may be willing to charge a negative interest rate. This isn't necessarily a problem for our story, but rather, makes the analysis more complicated. To avoid such complications, we adopt A4.

An important part of our model is the type of contracting that exists between the MFI and the social investors. The contracting governs the supply of subsidized funding. We consider two kinds of contracts. Both types of contract last for one period, and stipulate a quantity of debt and an interest rate. The contracts differ according to whether the investors are able to monitor and enforce the size

[9] In empirical studies, average loan size is often used as proxy for the level of outreach to the poor. See Armendariz and Szafarz (2011) for further discussion. Along these lines, an alternative interpretation for the big loans is a case where the MFI issues loans that encourage too much borrowing, which eliminates any social benefit. This of course would require an explanation for why borrowers take such loans, as examined in papers like Arnold and Booker (2013).

of loans that the MFI issues. In one contract we assume that the social investors can monitor and enforce the loan size selected by the MFI, at zero cost, and in the other contract, the investors cannot. We refer to the second type of contract as arms-length contracting.

2.2 The game

To organize the analysis, we use the following two-period game. The MFI itself aims to maximize his discounted expected payoff, over the two periods. We assume the MFI has a discount factor δ.

At the start of period one, the MFI is allocated capital X. Nature then assigns the MFI a type. The MFI is assigned type h with probability α, and type l with probability $1 - \alpha$. At this point, neither the MFI or the social investors observe type, but they do know α.

The MFI then leverages his capital by soliciting external debt from the social investors. To do this, the MFI proposes a quantity of funds and an interest rate. The social investors either accept the proposal and supply the funds, or turn it down.

Next, the MFI uses his total funding to issue small loans, and chooses whether to lend prudently or not. At the end of the first period, the MFI uses the loan repayment revenue to cover operating costs and settle external debt obligations. In the event that the loans are not repaid to the MFI, the MFI is classified as insolvent and the game ends here.

At the start of period two, the MFI is again seeded with capital funding X. At this point, the MFI learns its type, but the social investors do not. The MFI solicits external debt from the social investors and uses the funds to issue loans. In period two, the MFI has a choice whether to issue small loans or large loans. Simultaneously, the MFI also chooses whether to lend prudently or not. When the MFI makes these two decisions, we assume that they apply for the entire loan portfolio. Thus, there are effectively four different loan portfolios that the MFI can consider in period two. After loans are issued, the MFI realizes its loan repayments and uses this revenue to settle cost and debt obligations.

At this point the game ends. To help clarify the exact order of moves in the game, we provide a timeline in Figure 1.

One may notice there are few implicit assumptions associated with the structure of the game. A key one is that the external investors are able to costlessly enforce the MFI's repayment obligations for debt. In practice, it is likely that repayment incentives are based on reputation, which would require a repeated game. While this is beyond the scope our study, it is a promising avenue of research that might help explain some of the empirical observations in Garmaise and Natividad (2010).[10]

It is also worth emphasizing that our study is focused primarily on the interaction between the MFI and external investors. In a more generalized setting, it would be interesting to allow the MFI to alter the loan contract offered to borrowers. This would allow us to study how the interest rate r, as well as other contractual features might impact Δ_j.[11]

3. Equilibrium analysis

If neither type of MFI plans to issues small loans in period two, then the investors will not accept a rate below $i = i_m$. However, if the investors expect both types of MFI to issue small loans, then the investors will accept a lower rate. Since the investors do not observe the MFI's type, the investors calculate an expected social impact, namely $\alpha\Delta_h + (1-\alpha)\Delta_l$, or simply $\alpha\Delta_h$. This is the justification for the subsidy. In this case, the social investors are willing to distribute funds to the MFI at an interest rate as low as $i = i_m - \alpha\Delta_h$.

3.1 Portfolio choices in period two

Working backwards in period two, assume that the MFI has already raised external funding. At this point, the MFI must select both a loan size and whether to lend prudently or not. There are four

[10] In their study, Garmaise and Natividad (2010) find that positive evaluations of MFIs are associated with a reduction in the cost of financing for the MFI. This is most pronounced for young MFIs, suggesting that reputation matters.

[11] Some of these ideas are modeled in Ghosh and Van Tassel (2013).

possible portfolio choices to consider. Two of these options involve prudent lending, which we focus on first.

One option for the MFI is to offer small loans, under prudent lending. The MFI has a total capital allocation of X, which is used to fund part of each loan issued. If x denotes the amount of capital used per loan, then the MFI must use $1-x$ worth of debt funding per loan. Given this structure of funding, the MFI can then issue a total of $\frac{X}{x}$ small loans. This generates the MFI a financial payoff in period two of

$$\frac{X}{x}[1+r-c-(1+i)(1-x)]-(1+\rho)X \tag{1}$$

Note that in addition to this, the MFI also receives a social payoff of $\frac{X}{x}\Delta_j$. Since i never exceeds i_m, Assumption A1 implies that the MFI's combined payoff (financial plus social) is decreasing in x. Thus, the MFI prefers to use as little capital as possible per loan issued. However, as we explain below, the social investors will not supply debt unless they believe the MFI will be prudent, which in turn depends on how much capital the MFI uses per loan.

A second option for the MFI is to offer small loans, and not use prudent lending. If the MFI does not use prudent lending, then the MFI faces an expected financial payoff of[12]

$$\lambda\frac{X}{x}[1+r-(1+i)(1-x)]-(1+\rho)X \tag{2}$$

Hence, in terms of financial payoffs, the MFI prefers prudent lending practices as long as

[12]The expected social payoff is $\lambda\frac{X}{x}\Delta_j$.

$$x \geq \frac{c-(1-\lambda)(r-i)}{(1-\lambda)(1+i)} \tag{3}$$

Define $\overline{x}_{s,2}$ as the value of x where equation (3) binds. If the MFI uses at least $\overline{x}_{s,2}$ per loan, then according to the financial payoff, the MFI prefers prudent lending. In fact, one can confirm that if the MFI's combined payoffs (financial and social) are compared, the resulting lower bound on x is even lower.[13] However, in practice we seldom observe minimum capital requirements based on social benefit values. Rather, capital requirements are generally enforced by third parties, such as state regulators, who do not measure social benefit. To keep our study relevant, we choose to use capital requirements that are based on financial payoffs alone.[14] Furthermore, we assume that a state regulator enforces these requirements in every period of the game.

In period two, on a small loan, suppose the MFI uses capital $\overline{x}_{s,2}$ per loan. Plugging $\overline{x}_{s,2}$ into the MFI's payoff yields $X\pi_{s,2}(i,j)$, where

$$\pi_{s,2}(i,j) \equiv \frac{1+i}{c-(1-\lambda)(r-i)}[\lambda c+(1-\lambda)\Delta_j]-(1+\rho) \tag{4}$$

The other option for the MFI is to issue big loans. If the MFI allocates capital x_b to each big loan, then prudent lending generates the MFI a payoff in the second period equal to

$$\frac{X}{x_b}[(1+r)b-c-(1+i)(b-x_b)]-(1+\rho)X \tag{5}$$

Assumption A1 implies that this is decreasing in x_b and so, as before, the MFI prefers leverage. If the MFI issues big loans and is not prudent, then the expected payoff is

[13] In this case, it would be $x \geq \frac{c-(1-\lambda)(r-i+\Delta_j)}{(1-\lambda)(1+i)}$.

[14] Keep in mind that this is a conservative approach, in that the capital requirement will be higher than it actually needs be for the type h MFI

$$\lambda \frac{X}{x_b}[(1+r)b - (1+i)(b - x_b)] - (1+\rho)X \tag{6}$$

The MFI prefers prudent lending as long as

$$x_b \geq \frac{c - (1-\lambda)(r-i)b}{(1-\lambda)(1+i)} \tag{7}$$

We define $\overline{x}_{b,2}$ as the value of x_b where equation 7 binds.

Lemma 1. $0 < \overline{x}_{b,2} < \overline{x}_{s,2} < 1$.

This result follows directly from Assumption A2, implying that one advantage of bigger loans is that they have a lower capital requirement.

Plugging $\overline{x}_{b,2}$ into the MFI's payoff on a portfolio of big loans yields $X\pi_{b,2}(i)$, where[15]

$$\pi_{b,2}(i) \equiv \frac{1+i}{c - (1-\lambda)(r-i)b}\lambda c - (1+\rho) \tag{8}$$

Up to this point, we have focused on cases where the social investors expect the MFI to adopt prudent lending practices. It turns out that the alternative will not sustain intermediation, as indicated in the following Lemma.

Lemma 2. The MFI cannot borrow funds if the social investors expect the MFI to act imprudently.

[15] We have dropped the subscript on π indicating type because type does not influence the payoff when loans are big.

Proof. See appendix.

This result establishes the importance of minimum capital requirements. Without allocating the necessary capital per loan, the MFI will be unable to raise debt funding. Given the endowment of equity capital X, the minimum capital requirement implies that there is a limit on how much debt an MFI can raise, which in turn establishes a limit to how many loans an MFI can issue.

3.2 Benchmark

The main question in this paper is how subsidies influence microfinance lending. To establish a benchmark, we first analyse a case where the social investors do not care about social impact. This means that external debt cost $i = i_m$ and hence, there are no subsidies.

Consider period two, after the MFI has raised funding. A type j MFI has a choice between issuing small loans and large loans. The MFI prefers small loans if $\pi_{s,2}(i, j) > \pi_{b,2}(i)$, or

$$b \leq \frac{c\Delta_j + \lambda c(r - i)}{(r - i)(\lambda c + (1 - \lambda)\Delta_j)} \tag{9}$$

If we define $b_j(i)$ as the size of the big loan where this inequality binds, then it is straightforward to confirm the following.

Lemma 3. $b_h(i) > b_l(i) = 1$.

The low ability MFI always prefers to issue big loans, given a fixed interest rate for external debt. This is because the MFI generates no social return and the big loans are more profitable. In contrast, the high ability MFI generates both financial and social returns from small loans. Hence, big loans are optimal only when the financial payoff exceeds the combined payoff from a small loan portfolio.

In general, we find that the more effective the MFI is at generating a social impact from small loans, the higher b needs to be in order to make it an attractive asset choice.

For the benchmark, social investors charge $i = i_m$. This will be true for both periods. Denoting Π_m as the MFI's discounted expected profit over periods one and two, we have the following result.

Proposition 1. Assume social investors do not value social benefits. If $\Pi_m \geq 0$ and $b \leq b_h(i_m)$, then there is an equilibrium where in period two, the type h MFI issues small loans and the type l MFI issues big loans, and both types of MFI pay interest $i = i_m$ for external debt.

Proof. See appendix.

In the benchmark case, the cost of external funding is independent of the loan size issued by the MFI. This implies that in the second period the MFI specializes according to what it is good at. A high ability MFI continues to issue small loans because it finds that it is effective at this, whereas the low ability MFI switches to larger loan sizes that generate a higher financial return. In this case, the external supply of funds does not distort portfolio choice.

3.3 Subsidized external funding with monitoring

In practice, many microfinance organizations are subsidized. In this section of the paper we try to pin down how subsidies might impact the lending behavior of the MFI. A key part of this has to do with the kind of relationship that external investors have with the MFI.

To start, we assume that the social investors can contract on the asset choices of the MFI. This gives the investors the ability to link the interest rate directly to the type of loan that the MFI chooses to issue in period two.

We also assume that the MFI has access to market priced debt, in the event that he prefers it over subsidies. In practice, of course, institutions may not have such a simple choice. For example, a new microfinance institution may not have the luxury to choose between different kinds of funding. However, looking at a collection of 138 different MFIs, Garmaise and Natividad (2010) find that commercial funding as a percentage of total funds has increased dramatically between 1997 and 2008. This suggests that some MFIs do have a choice. In fact, studies like Mersland and Urgeghe (2013) and Cull et al. (2009) find that many institutions often use a variety of different kinds of funding, some subsidized, some not.

Consider a case where the MFI plans to issue small loans in period two regardless of his type. To denote the MFI's discounted payoff in the game under this lending plan, we use Π_A. In this case, the social investors demand an interest rate of $i = i_m - \alpha\Delta_h$ in both periods, which we denote as i_Δ. If a type j MFI raises external debt at the subsidized interest rate i_Δ in $t = 2$ and issues small loans, then the MFI earns $X\pi_{s,2}(i_\Delta, j)$ in period two.

The alternative for the MFI is to issue big loans. Of course this means that the MFI must pay $i = i_m$. Issuing big loans yields the MFI a second period payoff of $X\pi_{b,2}(i_m)$. In period two, the type j MFI prefers small loans if $\pi_{s,2}(i_\Delta, j) \geq \pi_{b,2}(i_m)$. Since $\pi_{s,2}(i_\Delta, h) > \pi_{s,2}(i_\Delta, l)$, both types of MFI prefer small loans if the type l MFI prefers small loans, or, if $\pi_{s,2}(i_\Delta, l) \geq \pi_{b,2}(i_m)$, or

$$b \leq b_1 \equiv \frac{(1-\lambda)(1+i_m)(r-i_m+\alpha\Delta_h) - \alpha\Delta_h c}{(1-\lambda)(1+i_m-\alpha\Delta_h)(r-i_m)} \tag{10}$$

This brings us to the following result.

Proposition 2. Suppose that $\Pi_A \geq 0$ and that social investors monitor the MFI's asset choices. There is a pooling equilibrium where both types of MFI issue small loans and pay interest $i = i_m - \alpha\Delta_h$ in both periods, as long as $b \leq b_1$.

Proof. See appendix.

This result describes an outcome where the MFI issues small loans in both periods. Since investors value the social benefits from (effective) small loans, the MFI is able to raise external debt at an interest rate below the market rate. This clearly benefits the MFI. However, there are strings attached to the subsidy. In this case, the investors require that the MFI issue small loans. Keep in mind that the MFI is still free to decline the subsidy and raise external debt at the market interest rate, as in Section 3.2. Hence, the equilibrium payoff to the MFI, Π_A, cannot be less than what the MFI earns in the absence of subsidies, namely Π_m.

This is not surprising. Cheap external funding raises the MFI's expected payoff. What is concerning though, is how subsidies distort lending behavior. In Proposition 2, the low ability MFI issues small loans instead of big loans. This portfolio choice is inefficient. If the low ability MFI deviated by issuing big loans, holding $i = i_\Delta$, then the MFI's second period expected profit would rise. Furthermore, the deviation would have no negative impact on social benefits.[16] This means that after the deviation, holding $i = i_\Delta$, the social investors still earn the same expected payoff.

Subsidies make financial intermediation more profitable, which makes intermediation more viable. However, we find that an MFI may distort its loan portfolio in an inefficient manner. The dilemma facing the low ability MFI is that if he did deviate and issue larger loans, the investors would withdraw funding, or at least raise i. Anticipating this, the MFI continues to issue small loans that have no social impact.

We document this kind of inefficient portfolio choice when $b \leq b_1$. Note that Assumption A2 implies that $b_1 > 1$. Also, note that if we set $\Delta_h = 0$, then it is easy to confirm that b_1 collapses to 1. This is because if there is no social benefit, then the MFI always prefers the big loan, as it is more profitable.

Lemma 4. $\frac{\partial}{\partial \alpha} b_1 > 0$ and $\frac{\partial}{\partial \lambda} b_1 < 0$

The positive sign on the first partial derivative follows directly from Assumption A2. This means that when social investors are more confident about the ability of the MFI to create social benefits,

[16] This is because we have set $\Delta_l = 0$. If instead, $\Delta_l > 0$, then a new tradeoff would emerge.

the larger the span of big loan opportunities that the MFI will decline. A higher α translates into a larger subsidy, which means the MFI will avoid issuing a larger b loan. With regard to the second partial, note that as λ rises, the capital requirement rises. This implies that the MFI uses less debt funding, which makes the subsidy less relevant in profit calculations. Thus, the MFI's preference for the big loans is higher.

The pooling behavior in the second period of the game arises because the low ability MFI prefers cheaper debt over issuing big loans. One way to interpret this outcome is in terms of different stages of growth for an MFI. The longer the MFI operates in the credit market, the more viable opportunities the MFI should have to pursue larger business loans. As the MFI collects information from its lending relationships, this information can be used to identify and supply loans to the most promising clients. Thus, we can argue that over time, b should rise for a given microfinance institution. In this sense, the pooling equilibrium might represent an early stage for a microfinance institution, when b is low.

Over time, as the institution learns more about the credit market, the institution should be capable of making more profitable types of loans. At some point, b will be high enough so that it becomes worthwhile for a low ability MFI to replace subsidized funding with market priced funding.[17] This allows the MFI to issue bigger loans. While the low ability MFI issues big loans, it can still be the case that the high ability MFI issues small loans. In this event, the MFI self-selects according to its realized type in the second period. To denote the MFI's discounted profit under this lending plan, we use Π_s.

Proposition 3. Suppose that $\Pi_s \geq 0$ and that social investors monitor the MFI's asset choices. There is a separating equilibrium where in the second period, the type h MFI issues small loans and pays $i = i_m - \Delta_h$, while the type l MFI issue big loans and pays $i = i_m$, as long as $b_2 \leq b \leq b_3$, where

[17] This argument fits well with the earlier point that in practice, microfinance instituions often do not have access to market priced debt. According to our results, the availability of market priced funding is only relevant if the institution has matured to the point where it can make larger and more profitable loans. A younger instituion, where b is low, has no incentive to raise debt at market price.

$b_2 \equiv \frac{(1-\lambda)(1+i_m)(r-i_m+\Delta_h)-\Delta_h c}{(1-\lambda)(r-i_m)(1+i_m-\Delta_h)}$ and $b_3 \equiv \frac{(1+i_m-\Delta_h)c[\lambda c+(1-\lambda)\Delta_h]-(1+i_m)\lambda c[c-(1-\lambda)(r-i_m+\Delta_h)]}{(1-\lambda)(r-i_m)(1+i_m-\Delta_h)[\lambda c+(1-\lambda)\Delta_h]}$.

Proof. See appendix.

In this case, we have an equilibrium outcome where the MFI specializes according to what it is good at. In order for this to occur, the larger loan opportunities in the second period must be sufficiently profitable to persuade the low ability MFI to stop using subsidies. This is exactly what b_2 denotes. When $b > b_2$, the low ability MFI declines the subsidized funding. However, at the same time, the opportunities must be not too profitable from the point of view of the high ability MFI. The high ability type must prefer to continue to rely on external subsidies in order to fund its small lending program. This is precisely what b_3 calculates. Namely, when $b \leq b_3$ the high ability MFI will not pursue the large loan opportunities.

The specialization in period two is exactly what occurred in the benchmark case. In this sense, efficient portfolio choice is restored under separation.

In Figure 2 we provide an example of the three cutoff values for b as a function of α. For a given value of α, the graph illustrates what parameter values of b will support the equilibria outcomes described in Propositions 2 and 3. In this case, the example relies on the following parameter values: $i_m = 0.06$, $r = 0.2$, $\Delta_h = 0.05$, $c = 0.1$ and $\lambda = 0.76$.

3.4 Arms-length subsidized debt

We now assume that the social investors cannot contract on the MFI's asset choices. This means that in the second period, after raising external funds, the MFI is free to choose either loan size. At a given interest rate on external debt, the MFI compares its second period payoffs according to loan size. As we calculated earlier, the type l MFI always issues big loans, whereas the high ability MFI issues big loans only if $b > b_h(i)$.

We now examine a strategy where the high ability MFI plans to issue small loans and the low ability MFI plans to issue big loans. This means that the social investors expect a social impact of $\alpha\Delta_h$ and hence, demand $i = i_m - \alpha\Delta_h$. We denote the MFI's discounted profit under this lending plan as Π_B.

Proposition 4. Let $\Pi_B \geq 0$ and assume social investors cannot monitor the MFI's asset choices. There is a pooling equilibrium where both types of MFI pay interest $i = i_m - \alpha\Delta_h$ in each period, the type h MFI issues small loans, and the type l MFI issues big loans, as long as $b \leq b_h(i_\Delta)$.

Proof. See appendix.

Once an MFI obtains its external debt, the MFI faces a choice about loan size. In this section of the paper, the interest rate on debt is not conditional on the MFI's choice of loan size. Consequently, as explained in Section 3.1, the low ability MFI will issue big loans. On the other hand, the high ability MFI continues with its small lending program, as long as the bigger loans opportunities are not too profitable, or $b \leq b_h(i_\Delta)$.

One can easily confirm that $\Pi_B \geq \Pi_A$. The intuition is straightforward. The interest rate used for both of these profit calculations is the same, namely i_Δ. What is different is that under the arms-length contracting, the type l MFI issues big loans in $t = 2$. This raises the expected profit the MFI can earn in period two. Thus, the leniency in the external debt contract makes financial intermediation more lucrative.

In the second period, under the arms-length contract, the two types of MFI specialize according to what they are good at. While an MFI may indeed end up using subsidized funding in a manner for which it was not intended, this has minimal, or in our model, zero negative implications regarding social impact. The potential downside of a funding environment where investors cannot enforce asset selection turns out to be minimized through the endogenous behavior of the MFIs themselves.

Under arms-length contracting, we assume investors do not monitor loan size. We do assume however, as pointed out in Section 3.1, that regulators ensure that capital requirements are satisfied. Given that arms-length funding has certain advantages, one could make the argument that social investors might be wise to put more focus on monitoring capital requirements and less focus on the size of loans issued by the MFI.

3.5 Dynamic implications

In our model, the microfinance institution calculates a discounted expected payoff from lending over the two periods. The second period of the game is used to represent the case where due to prior experience, the institution has a choice of loan opportunities. During this phase, the type of funding available to the institution and the contracting that governs it has important implications for period one.

In the pooling equilibrium described by Proposition 2, the microfinance lender issues small loans in the second period regardless of type. This distortion of the loan portfolio has implications for the capital requirement in period one. This is because the requirement, namely $\overline{x}_{A,1}$, is a function of the second period's expected profit earned by the institution.[18] It is interesting to compare $\overline{x}_{A,1}$ with the requirements calculated for the other equilibria. In particular, in Propositions 3 and 4, the first period capital requirement is identical to the capital calculated for the benchmark analysis, namely $\overline{x}_{m,1}$.

Lemma 5. $\overline{x}_{m,1} < \overline{x}_{A,1}$.

This holds because $\pi_{b,2}(i_m) > \pi_{s,2}(i_m, l)$. The expected financial profit earned by the MFI in period two in Propositions 3 and 4 is higher than the financial profit earned in Proposition 2. Recall that in Propositions 3 and 4, the low quality MFI issues big loans in period two, which yields a higher financial profit than small loans. The higher expected profit turns out to help alleviate the moral

[18] The value of all first period capital requirements are located in the appendix.

hazard problem involving prudent lending in period one and so, the minimum necessary capital is less.

While achieving separation in Proposition 3 requires a minimum cutoff for b, namely b_2, there is no such restriction for the arms-length contract. That is, under arms-length contracting, we support the equilibrium at b values arbitrarily close to 1. Given this, it is worth comparing the performance of the pooling equilibria described under the monitoring contract with the pooling that occurs under the arms-length contract.

Proposition 5. Suppose that $b \leq \min\{b_1, b_h(i_\Delta)\}$. The equilibrium under arms-length contracting is characterized by a higher expected social impact from microfinance lending than the equilibrium under monitoring.

This result can be confirmed by comparing the first period capital requirements under Propositions 2 and 4. When $b \leq \min\{b_1, b_h(i_\Delta)\}$, we can support either equilibrium, as described in the two propositions. Since $\overline{x}_{m,1} < \overline{x}_{A,1}$, arms-length contracting allows the MFI to issue a relatively larger number of loans in the first period. A larger loan portfolio translates to a larger expected social impact. Thus, when an MFI can raise external debt from social investors who lack the ability to monitor the MFI's loan size choices, the MFI can increase his first period outreach. This means that the total expected social impact from microfinance is actually higher than what it would be if the MFI was funded by a single donor, who closely monitored the asset choices of the MFI.[19] This offers one argument for how more distant, less informed social investors can be compatible with poverty reduction.

With less strings attached to the subsidized funding, the MFI views its intermediation activities over the long-run as a more profitable and rewarding activity. The intuition is straightforward. If the

[19] If we alter our assumption about Δ_l, so that $\Delta_l > 0$, then there is an additional social cost to consider. Namely, when the low quality MFI switches to big loans under arms-length funding, a positive social impact is lost. This would then work towards countering the advantage of arms-length contracting.

social outcome from lending turns out to be disappointing, then the MFI can alter its course and use its experience to pursue more profitable lending opportunities. The higher expected earnings in period two lead to more prudent behavior, and reduce the necessary capital requirement for microfinance. Not only does this lead to more outreach and a higher expected social impact in the first period, but it also raises the return on capital. Recall that $\Pi_B \geq \Pi_A$. In this sense, one can say that arms-length contracting for external debt can raise the charter value of the microfinance institution.

4. Conclusion

We have studied a two-period framework where a microfinance institution receives subsidized funding from imperfectly informed external investors who value social impact. In a pooling equilibrium, the microfinance institution may choose to solicit external subsidies even when its small lending program generates little or no social benefit. We identify two factors that contribute to this outcome. One is large subsidies. The cheaper the external funding, the higher the incentive for an institution to maintain an ineffective lending portfolio. This can be true even if the institution itself values social impact. Secondly, less opportunities to issue larger, more profitable loans makes it more likely that a microfinance institution will continue to use subsidies ineffectively.

On the other hand, when these two factors are diminished, we identify a separating equilibrium. In this case, the microfinance institutions specialize according to what they are good at. Institutions with effective small lending programs continue to issue such loans using subsidized funding, while other institutions use market priced debt to fund more profitable types of loans.

Another contribution of our research is related to how microfinance funding has been changing over the years. The role of international social investors has become more significant relative to traditional types of microfinance funding. Some analysts have voiced concern that this may lead to less emphasis on poverty reduction. While this is clearly a difficult question to answer, our analysis offers a few insights. We find that leniency in debt contracts can indeed be associated with an outcome where a lender uses subsidies in a manner for which they were not intended. However, the types of microfinance institutions that choose to do this are the same institutions that had little or no social impact to begin with. Thus, we argue that arms-length investors do not necessarily reduce social impact. Furthermore, leniency in debt contracts raises the discounted, expected charter value of the microfinance institution. This can lower capital requirements and potentially allow an institution to achieve greater outreach and more poverty reduction.

In this study, we have focused on how a microfinance institution may encounter larger, more profitable lending opportunities over time. An alternative approach could be to consider how an institution might improve its social impact over time. One way to model this might be to allow social impacts to converge over time, which would then diminish the relevance of asymmetric information. These are interesting questions but beyond the reach of our paper, so we leave them to future research.

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Appendix

Proof of Lemma 2. Suppose the MFI is not prudent. If the MFI issues small loans, then the investors expect a payoff of $\lambda(1+i+\alpha\Delta_h)$. Hence, the investors demand a rate of $i = \frac{1+i_m}{\lambda} - \alpha\Delta_h - 1$. If the MFI issues big loans, then the investors demand $i = \frac{1+i_m}{\lambda} - 1$. On small loans a type j MFI earns

$$\lambda \frac{X}{x}(1 + r - (1+i)(1-x) + \Delta_j) - (1+\rho)X \tag{11}$$

and on big loans the MFI earns

$$\lambda \frac{X}{x_b}[(1+r)b - (1+i)(b - x_b)] - (1+\rho)X \tag{12}$$

Taking the partial derivatives of these two payoffs with respect to x (x_b), at the interest rates described above, one can confirm that both partials are negative. This means that the MFI's profit is highest when $x = 0$, for a given i. Hence, say the MFI plans to use $x = 0$ per loan, and just fund the loans with debt. Since the MFI is not prudent, the investors require a risk premium, as described by the two interest rates given above. If we plug these rates into the MFI's payoff, the payoffs are negative, due to Assumption A4. This means that unless the MFI satisfies the capital requirement, microfinance is not worthwhile. QED

Proof of Proposition 1. In period two, the MFI specializes by loan size according to his type. A type h MFI issues small loans because $b \leq b_h(i_m)$ and the type l MFI issues big loans. Note that regardless of loan size, $i = i_m$. Furthermore, from Section 3.1 we know the MFI will choose prudent lending practices in period two.

In period one the MFI has a choice whether to use prudent lending practices or not. If the MFI funds the \$1 loans using capital x_1 and is prudent, then the MFI's discounted profit in the game is

$$\frac{X}{x_1}\left[1+r-c-(1+i_m)(1-x_1)+\alpha\Delta_h\right]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}\left(i_m,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right] \tag{13}$$

Alternatively, if the MFI is not prudent in period one, then its profit is

$$\lambda\frac{X}{x_1}\left[1+r-(1+i)(1-x_1)+\alpha\Delta_h\right]-(1+\rho)X+\lambda\delta X\left[\alpha\pi_{s,2}\left(i_m,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right] \tag{14}$$

The MFI prefers to use prudent lending practices in the first period as long as

$$x_1 \geq \frac{c-(1-\lambda)(r-i_m+\alpha\Delta_h)}{(1-\lambda)\left(1+i_m+\delta\left[\alpha\pi_{s,2}\left(i_m,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right]\right)} \tag{15}$$

As mentioned earlier, we do not use social benefits when calculating capital requirements. Since $\frac{\partial}{\partial\Delta}\pi_{s,2}\left(i_m,h\right)>0$ and $\frac{\partial}{\partial\Delta}\pi_{b,2}\left(i_m\right)=0$, it is straightforward to confirm that

$$\frac{\partial}{\partial\Delta_h}\frac{c-(1-\lambda)(r-i_m+\alpha\Delta_h)}{(1-\lambda)\left(1+i_m+\delta\left[\alpha\pi_{s,2}\left(i_m,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right]\right)}<0 \tag{16}$$

This means that the first period capital requirement is decreasing in Δ_h, just as it was in period two. Hence, if we use $\Delta_h=0$ to compute the capital requirement, then this is sufficient to induce both types of MFI to be prudent. We define the first period capital requirement as[20]

[20] Note that since we use $\Delta_h=0$ to compute the capital requirement, $\pi_{s,2}\left(i_m,h\right)$ collapses to the equivalent of $\pi_{s,2}\left(i_m,l\right)$.

$$\overline{x}_{m,1} \equiv \frac{c-(1-\lambda)(r-i_m)}{(1-\lambda)[1+i_m+\delta(\alpha\pi_{s,2}\left(i_m,l\right)+(1-\alpha)\pi_{b,2}\left(i_m\right))]} \tag{17}$$

Plugging $\overline{x}_{m,1}$ into the MFI's discounted payoff under prudent lending yields

$$\Pi_m = \frac{X}{\overline{x}_{m,1}}[1+r-c-(1+i_m)(1-\overline{x}_{m,1})+\alpha\Delta_h]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}\left(i_m,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right] \tag{18}$$

As long as $\Pi_m \geq 0$, the MFI will choose to intermediate and issue loans. QED

Proof of Proposition 2. In period two, since $b \leq b_1$, the MFI chooses small loans and earns a profit of either $X\pi_{s,2}\left(i_\Delta,h\right)$ or $X\pi_{s,2}\left(i_\Delta,l\right)$, depending on his type. Also, as explained in Section 3.1, the MFI is subject to minimum capital requirements and hence, will use prudent lending practices in period two.

In period one, if the MFI acts prudently, then the MFI's discounted expected profit in the game is

$$\frac{X}{x_1}[1+r-c-(1+i_\Delta)(1-x_1)+\alpha\Delta_h]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}\left(i_\Delta,h\right)+(1-\alpha)\pi_{s,2}\left(i_\Delta,l\right)\right] \tag{19}$$

In contrast, imprudent lending in the first period yields

$$\frac{X}{x_1}\lambda[1+r-(1+i_\Delta)(1-x_1)+\alpha\Delta_h]-(1+\rho)X+\lambda\delta X\left[\alpha\pi_{s,2}\left(i_\Delta,h\right)+(1-\alpha)\pi_{s,2}\left(i_\Delta,l\right)\right] \tag{20}$$

The MFI will choose prudent lending in period one when subject to the following the capital requirement:

$$\overline{x}_{A,1} \equiv \frac{c-(1-\lambda)(r-i_m)}{(1-\lambda)[1+i_m+\delta\pi_{s,2}(i_m,l)]} \tag{21}$$

Plugging $\overline{x}_{A,1}$ into the MFI's discounted payoff under prudent lending yields

$$\Pi_A = \frac{X}{\overline{x}_{A,1}}[1+r-c-(1+i_\Delta)(1-\overline{x}_{A,1})+\alpha\Delta_h]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}(i_\Delta,h)+(1-\alpha)\pi_{s,2}(i_\Delta,l)\right] \tag{22}$$

Thus, as long as $\Pi_A \geq 0$ the MFI prefers to intermediate and issue loans. QED

Proof of Proposition 3. At the interest rate $i = i_m - \Delta_h$, the type h MFI earns $X\pi_{s,2}(i,h)$. If this MFI deviates, and pays $i = i_m$, the MFI earns $X\pi_{b,2}(i_m)$. Hence, for incentive compatibility we require that $\pi_{s,2}(i,h) \geq \pi_{b,2}(i_m)$, or

$$b \leq \frac{(1+i_m-\Delta_h)c[\lambda c+(1-\lambda)\Delta_h]-(1+i_m)\lambda c[c-(1-\lambda)(r-i_m+\Delta_h)]}{(1-\lambda)(r-i_m)(1+i_m-\Delta_h)[\lambda c+(1-\lambda)\Delta_h]} \tag{23}$$

The type l MFI issue big loans and funds the loans at $i = i_m$. This gives the MFI a profit of $\pi_{b,2}(i_m)$, which is better than issuing small loans as long as $\pi_{b,2}(i_m) \geq \pi_{s,2}(i,l)$, where $i = i_m - \Delta_h$, or

$$b \geq \frac{(1-\lambda)(1+i_m)(r-i_m+\Delta_h)-\Delta_h c}{(1-\lambda)(r-i_m)(1+i_m-\Delta_h)} \tag{24}$$

In the first period, prudent lending yields

$$\frac{X}{x_1}[1+r-c-(1+i_\Delta)(1-x_1)+\alpha\Delta_h]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}\left(i_m-\Delta_h,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right] \quad (25)$$

whereas imprudent lending yields

$$\frac{X}{x_1}(\lambda[1+r-(1+i_\Delta)(1-x_1)+\alpha\Delta_h]-(1+\rho)X+\lambda\delta X\left[\alpha\pi_{s,2}\left(i_m-\Delta_h,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right] \quad (26)$$

Thus, to induce prudent behavior, we require a capital of

$$x \geq \frac{c-(1-\lambda)(r-i_m)}{(1-\lambda)[1+i_m+\delta(\alpha\pi_{s,2}\left(i_m,l\right)+(1-\alpha)\pi_{b,2}\left(i_m\right))]} \quad (27)$$

Note that this is identical to $\overline{x}_{m,1}$, as calculated in the benchmark case in Section 3.2. Plugging this into the MFI's discounted profit yields

$$\Pi_s=\frac{X}{\overline{x}_{m,1}}[1+r-c-(1+i_\Delta)(1-\overline{x}_{m,1})+\alpha\Delta_h]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}\left(i_m-\Delta_h,h\right)+(1-\alpha)\pi_{b,2}\left(i_m\right)\right] \quad (28)$$

As long as $\Pi_s \geq 0$, the intermediation is funded.

Finally, let us confirm how b_1, b_2 and b_3 compare. First, $b_1 \leq b_2$ if

$$\Delta_h c(1-\alpha)(1+i_m) \leq (1-\lambda)(1+i_m)\left[(r-i_m+\alpha\Delta_h)\Delta_h-(r-i_m+\Delta_h)\alpha\Delta_h\right] \quad (29)$$

$$\Delta_h c(1-\alpha) \leq (1-\lambda)(1-\alpha)\left[\Delta_h(1+i_m)+\Delta_h(r-i_m)\right] \tag{30}$$

This binds if $\alpha = 1$, otherwise for $\alpha < 1$ we have $c < (1-\lambda)(1+r)$, which holds due to Assumption A2. Thus, $b_1 < b_2$ when $\alpha < 1$. Second, $b_2 < b_3$ if

$$(1-\lambda)(1+i_m)(r-i_m+\Delta_h)\left[\lambda c+(1-\lambda)\Delta_h\right] < (1-\lambda)(1+i_m)\left[c\Delta_h+\lambda c(r-i_m+\Delta_h)\right] \tag{31}$$

$$(1-\lambda)(r-i_m+\Delta_h) < c \tag{32}$$

Since A2 states that $(1-\lambda)rb < c$, $b_2 < b_3$ if $r - i_m + \Delta_h < rb$, which holds because A4 states that $i_m \geq \Delta_h$. QED

Proof of Proposition 4. In the second period, the MFI elects to use prudent lending, as explained in Section 3.1. Furthermore, since the social investors cannot observe the choice of loan size by the MFI, once the MFI has its funds, the low ability MFI is better off issuing large loans. Also, given that $b \leq b_h(i_\Delta)$, the high ability MFI has no incentive to deviate from issuing small loans.

In the first period, the MFI's expected payoff in the game from acting prudently is

$$\frac{X}{x_1}\left[1+r-c-(1+i_\Delta)(1-x_1)+\alpha\Delta_h\right]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}(i_\Delta,h)+(1-\alpha)\pi_{b,2}(i_\Delta)\right] \tag{33}$$

Alternatively, imprudent lending in the first period yields

$$\frac{X}{x_1}(\lambda[1+r-(1+i_\Delta)(1-x_1)+\alpha\Delta_h]-(1+\rho)X+\lambda\delta X\left[\alpha\pi_{s,2}\left(i_\Delta,h\right)+(1-\alpha)\pi_{b,2}\left(i_\Delta\right)\right] \tag{34}$$

To calculate the first period capital requirement, we use $\Delta_j = 0$, which yields a requirement of $\overline{x}_{m,1}$, exactly as in the benchmark case. If we plug $\overline{x}_{m,1}$ into the MFI's discounted expected profit we have

$$\Pi_B = \frac{X}{\overline{x}_{B,1}}[1+r-c-(1+i_\Delta)(1-\overline{x}_{B,1})+\alpha\Delta_h]-(1+\rho)X+\delta X\left[\alpha\pi_{s,2}\left(i_\Delta,h\right)+(1-\alpha)\pi_{b,2}\left(i_\Delta\right)\right] \tag{35}$$

Consequently, as long as Π_B the MFI prefers to intermediate rather than not. QED

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