Multiple borrowing by small firms under asymmetric information

Eric Van Tassel*

August 28, 2014

Abstract

An entrepreneur planning a risky expansion of his business project may prefer to fund the expansion by soliciting several loans from different banks. While this is inefficient due to the duplication of screening and monitoring costs, it works to the entrepreneur's advantage if he can lower his risk premium by deceiving the banks about his investment intentions. When entrepreneurs are able to multiple borrow in equilibrium, it take places within a pooling contract, characterized by cross-subsidization. This kind of behavior in the credit market leads to high interest rates and in some cases, market failure due to adverse selection.

JEL codes: G21; D82; D86

Keywords: multiple loans, asymmetric information, bank competition, adverse selection

* *Department of Economics, Florida Atlantic University, 777 Glades Road, Boca Raton, FL 33431. Email vantasse@fau.edu Phone (561) 297-4176 Fax (561) 297-2542*

1. Introduction

A number of empirical studies find that small firms frequently choose to finance their business investments by taking out several loans from different banks. For example, based on a survey involving more than 5,000 small U.S. firms who have at least one lender, Guiso and Minetti (2010) report that 49% of the firms rely on two or more lending institutions. In other countries, the numbers are higher. Comparing small firms in the U.S. and Italy, Detragiache et al. (2000) finds that on average a U.S. firm has 2.3 banking relationships while an Italian firm has 12.3.[1] Even in microfinance markets, with smaller loan sizes yet, policy papers like Schicks and Rosenberg (2011) point out that multiple borrowing is increasingly common, especially in more mature credit markets. Examining microfinance markets in Nicaragua, Morocco and Bosnia-Herzegovina, Chen et al. (2010) report incidences of multiple borrowing between 20 and 40 percent of active borrowers.[2]

What is surprising about this evidence is that small firms are usually thought of as being informationally opaque.[3] To mitigate the information asymmetries, a bank must take a series of prudent and costly steps to ensure that the loan is repaid. A good part of these costs are independent of the actual size of the loan. The implication is that the screening and monitoring cost for a small sized loan necessitates a high mark up on the interest rate. Microfinance provides a good example of this. Looking at 346 institutions that issue microloans, Cull et al. (2009) find that the median bank, holding larger sized loans on average, spends 12 cents on operating costs per dollar of loan, whereas the median NGO, holding smaller loan sizes, spends 26 cents. Controlling for a number of different factors, the authors confirm that the institutions that issue the smallest loans on average, are the same institutions that face the highest cost per unit lent and furthermore, charge their customers the highest interest rates.

Given this cost based argument in favor of an exclusive banking relationship, as explained eloquently by Diamond (1984), the empirical evidence on multiple borrowing presents a puzzle. The existing literature has proposed

[1] Similar evidence is presented by Ongena and Smith (2000) and Farinha and Santos (2002).

[2] For more examples, see studies like McIntosh et al. (2005), Chua and Tiongson (2012), Frisancho (2012) and Khandket and Samad (2014).

[3] See Peterson and Rajan (1994) and Berger and Udell (2002).

several explanations, as we discuss below. Our paper attempts to add to this literature. For our study, we focus on small and micro sized firms, where the small loan size implies that transaction costs have a more notable impact on the interest rate. We use a one period model to examine the behavior of a population of entrepreneurs in a credit market where banks are imperfectly informed about the entrepreneurs' investment opportunities. We assume that a subset of the population has an opportunity to scale up the size of their existing business, though at a higher average level of risk. These entrepreneurs face a choice about how to finance their business expansion. One option is to take out a single large loan and the other option is to take out several small loans. When making this choice the entrepreneur compares the interest rates.

Two basic factors help shape the interest rates on the different loan sizes. One is the transaction cost associated with issuing a loan, which dictates an inverse relationship between the interest rate and loan size, all else equal. The other factor is a risk premium that banks charge the (average) customer who plans to scale up his business project. As opposed to the transaction cost, the risk premium implies a positive relationship between the interest rate and loan size, all else equal. In general, the net impact of these two factors can go either way.

When the risk premium dominates, we uncover a clear rational for multiple borrowing. If entrepreneurs choose to take out several small loans, the banks anticipate this and add a risk premium to the small loans. While this drives the rate up, there are also entrepreneurs who do not multiple borrow. These entrepreneurs cross-subsidize the entrepreneurs that do. The subsidy minimizes the effect of the risk premium. This provides the justification for multiple borrowing. Basically, the subsidy in the pooling contract used to issue small loans is sufficient to outweigh the disadvantages associated with high transaction costs. It turns out that not only is this behavior inefficient, in that it leads to excessive transaction costs, but in some cases it can aggravate adverse selection to the point where the entire credit market for small loans collapses.

A key assumption to our argument is that a small firm, on average faces additional risk if he chooses to scale up the size of his existing business project. We model this by assuming that one type of entrepreneur can scale up at no additional risk, while the other type faces a higher probability of failure. In an extension of our basic model, we then endow banks with the capacity to further screen the firms who apply for a larger loan size.

This allows the banks to identify (some of) the entrepreneurs who can scale up at low risk. A side effect of the screening is that a relatively risky group of firms is denied low interest rate loans. These entrepreneurs, should they elect to proceed with the business expansion, have strong incentives to multiple borrow. One of our results then, is that more intense bank screening can in fact lead to a higher incidence of multiple borrowing in the credit market.

The link between bank screening and multiple borrowing has some similarities with papers like Broecker (1990), Riordan (1993), Marquez (2002) and Direr (2008). In these models, a bank either screens clients or gleans private information during a lending relationship. While this gives the bank an information advantage, it has negative externalities on the rest of credit market. The externalities can affect the average quality of borrower, market interest rates, and the degree to which other banks must also screen. There is a similar mechanism at work in our paper. The entrepreneurs who are denied a low priced loan for their business expansion, as a result of bank screening, turn to the small loan market and essentially commit moral hazard by multiple borrowing.

In the existing literature, there are a few different explanations for why entrepreneurs might choose to have more than one lending relationship with a bank. One is based on the idea that a bank acquires private information about its borrowers over the course of a lending relationship. As argued by Sharpe (1990), Rajan (1992), Hubert and Schafer (2002), Von Thadden (2004) and others, a borrower can temper information based rent extraction by entering into multiple lending relationships. Another explanation is offered by Bolton and Sharfstein (1996) and Dewatripont and Maskin (1995), who point out that when firms find themselves in trouble, they naturally have an incentive to renegotiate the terms of their debt. The problem is that ex ante, the possibility of future renegotiation has negative implications on the initial contracting options. To alleviate this problem, the firm can enter into several lending relationships, making ex post renegotiation a trickier prospect. Carletti (2004) proposes a third possibility, using a model where a firm finds that the degree of bank monitoring within an exclusive lending relationship to be excessive. To counter this, the firm harnesses a free rider effect by relying on several different banks to obtain his funding.

Other explanations for multiple borrowing have more to do with limitations inherent to the bank. Carletti et al. (2007) argue that if a bank is unable to achieve a satisfactory level of diversification for its loan portfolio, then the bank may deliberately decrease the size of its loans in order to issue a larger number of smaller sized loans. Also, Detragiache et al. (2000) point out that a firm may avoid an exclusive banking relationship out of a concern about the bank's future ability to service the firm's demands. For example, the bank might experience a temporary liquidity shortage.

Another strand of literature related to our work is devoted to understanding multiple borrowing in the context of consumer credit. Papers such as Bizer and DeMarzo (1992), Kahn and Mookerjee (1998), Parlour and Rajan (2001) and Bisin and Guaitoli (2004) examine the borrowing decisions of a single agent who is supplied loans by multiple banks. In these models, the focus is on debt dilution, where the agent decreases the value of debt issued in the past by continuing to issue new debt. The literature explores a variety of different ways that this moral hazard problem can be alleviated, such by having the agent pre commit to only taking a specific amount of debt, by establishing system of seniority among the different lenders, or by rationing the supply of loans. While we also associate higher amounts of credit with higher (average) risk, our model is built differently. For one, we have a population of heterogenous entrepreneurs. Thus, while some entrepreneurs take on a larger amount of debt, others don't, which generates market outcomes where pooling contracts can promote multiple borrowing. Also, unlike the consumer credit literature, our emphasis on transaction costs generates a rational for a bank to issue a single, large sized loan as a potentially profitable alternative to multiple, smaller sized loans.[4]

There are also a few models that document multiple borrowing in credit markets applicable to the developing countries. Looking at a microfinance market, McIntosh and Wydick (2005) predict that increased entry and/or competition between banks, assuming no information sharing, can lead to multiple borrowing by a set of myopic agents. The agent's decision to take our multiple loans is similar in design to the consumer credit models mentioned above, in that additional loans worsens repayment incentives, aggravating a moral hazard problem.

[4] Livshits et al. (2011) introduce a transaction cost into a consumer credit model, though they assume the transaction cost is incurred for each type of contract, rather than per contract issued. In this setting, the authors find that the bank may choose to bundle several different borrower types under one loan contract as a means of keeping transaction costs low.

Jain (1999), Anderson and Malchow-Moller (2006) and Gine (2011) explain how multiple borrowing can result when two different kinds of lenders, namely a bank and moneylender, service the same population of borrowers. Assuming banks have lower costs and moneylenders have better information, these models explain how multiple borrowing emerges as a way of optimizing on the relative strenghs of the different kinds of lenders.

Of the papers focused on developing countries, our model is probably closest to McIntosh and Wydick (2005), in that we assume banks are identical and that the credit market is competitive. Our model takes a somewhat different perspective, by assuming that demand for larger aggregate debt comes from agents who plan larger business projects, all of which are efficient. In this sense, demand for larger sized loans is legitimate, whereas in McIntosh and Wydick (2005), as well as most of the consumer credit literature, large aggregate debt is inefficient. Another key finding in our paper is that when entrepreneurs engage in multiple borrowing in equilibrium, banks offer different interest rates on small loans. That is, we find that bank lending strategies under Bertrand competition are not symmetric. It turns out that an important implication of this, unlike McIntosh and Wydick (2005), is that introducing information sharing between banks does not help in avoiding multiple borrowing.

We have organized the paper as follows. In Section 2 we introduce the basic model of the credit market. The model includes a population of entrepreneurs who have varied investment opportunities and several banks who service the entrepreneurs. We assume there is Bertrand price competition between the banks. In Section 3 of the paper we study equilibrium behavior in the credit market. This analysis is divided into two separate subsections. In Section 3.1, we examine a case where entrepreneurs have no incentive to multiple borrow. In this case, equilibrium is efficient. In Section 3.2 incentives change, and we document an equilibrium outcome where entrepreneurs take out several different loans in equilibrium. Next, in Section 4 we study adverse selection. Here, we explain how the credit market may fail due to the possibility of multiple borrowing by entrepreneurs. In Section 5 we explore three different extensions of the basic model. It is here where we establish a link between bank screening and multiple borrowing. Finally, in Section 6 we have the conclusion.

2. The Model

Consider a one period model with n risk neutral entrepreneurs, where n is a large number. Each entrepreneur faces a choice between investing in a production project or earning a fixed (wage) income of $w \geq 0$. The project requires a \$1 investment and generates revenue R with probability p and revenue 0 with probability $1 - p$. Fraction λ of the n entrepreneurs have a third alternative. These entrepreneurs can choose to scale up their production projects by investing \$2 instead of \$1. The outcome of the larger project depends on the entrepreneur's type. There are two types of entrepreneur. A *high ability* type generates revenue $2R$ with probability p, and 0 otherwise, while a *low ability* type generates revenue $2R$ with probability p_l, and 0 otherwise. We assume that $p > p_l$.[5] Within the set of λn entrepreneurs who can scale up their projects, fraction α are type h and fraction $1 - \alpha$ are type l, where $0 < \alpha < 1$.

All entrepreneurs are endowed with zero wealth. To invest in a project the entrepreneur must obtain a loan from a bank. Loans are distributed according to contracts that specify a loan size and an interest rate. All loan contracts are limited liability. There are $m \geq 3$ banks. The banks themselves have access to an unlimited supply of funds at an interest rate of zero. A banks acts as an intermediary, transforming funding into loans for the entrepreneurs, with the objective of maximizing expected profit. Every loan issued by a bank costs the bank c, regardless of loan size, and $c > 0$.

Assumption A1 $pR > 1 + c + w$

Assumption A2 $p_l 2R > pR + 1 + c$

Assumption A1 implies that it is efficient for an entrepreneur to invest in the \$1 project rather than earn a wage. Assumption A2 implies that regardless of the entrepreneur's type, it is efficient for an entrepreneur to choose his \$2 project over his \$1 project, even if the entrepreneur funds his \$2 project using two separate \$1 loans.

[5] Note that if a low ability agent chooses the \$1 project, then the probability of success is p, not p_l.

Assumption A3 $p(1 + \frac{1}{2}c) > p_l(1 + c)$

While Assumption A3 isn't necessary for our main results, it keeps things interesting by restricting our attention to parameter values where the two types of entrepreneur are sufficiently different in terms of probabilities.

We assume that ex ante, banks do not know which entrepreneurs have the option to scale up their projects and which do not. Also, in the event that an entrepreneur reveals that he has an opportunity to scale up, by say, demanding a larger loan size, we assume that the banks are unable to observe the entrepreneur's type. Under this context of asymmetric information, we allow entrepreneurs to mislead banks about their intentions with regard to a \$1 loan. Specifically, an entrepreneur who has access to a \$2 project can choose to take out two separate \$1 loans, from different banks, by claiming that he intends to invest in his \$1 project. After receiving the two \$1 loans, the entrepreneur then uses the combined funds to invest in his \$2 project.

The game works as follows. Each bank simultaneously announces a set of loan contracts that are available to the entrepreneurs. Next, banks observe all the contracts that were offered, and then the banks have the option to react by offering additional contracts.[6] If one or more banks add contracts, then banks again observe the offers and are allowed to add additional contracts. This continues until all banks elect not to offer any additional contracts. At no point may a bank withdraw a contract that has already been offered. Also, we assume that when a bank is indifferent between lending and not lending, he always elects to offer a contract rather than not offer one.

Once banks have finished offering loan contracts, the entrepreneurs observe all offers and select loan contracts. Entrepreneurs who have access to the \$2 project are allowed to either select one \$2 loan, two \$1 loans, or one \$1 loan. When an entrepreneur demands two \$1 loans, we say the entrepreneur is *double borrowing*. Entrepreneurs who do not have access to the \$2 project may only request one \$1 loan. After the loans are issued to the entrepreneurs, the entrepreneurs invest in their projects. Finally, project revenue is realized and the proceeds are used to repay the loans. At this point, the game ends.

[6] We allow banks to react in order to deal with an existence problem. The intuition is explained by Riley (1979).

3. Competition in the Credit Market

In the credit market there are $(1-\lambda)n$ entrepreneurs who only have access to the \$1 project. These entrepreneurs require a \$1 loan. If these entrepreneurs select a \$1 loan, and no other entrepreneurs do, then competition between the banks leads to a competitive interest rate of $r_1 = \frac{1}{p}(1+c) - 1$. This leaves the λn entrepreneurs with access to both project sizes. In this subset of entrepreneurs, there are $\alpha\lambda n$ high ability entrepreneurs and $(1-\alpha)\lambda n$ low ability entrepreneurs. If both types of entrepreneur demand the same \$2 loan, then a bank can afford a pooling interest rate r, where

(1) $$[\alpha p + (1-\alpha)p_l]\, 2(1+r) - 2 - c = 0, \text{ or}$$

$$r_2 = \tfrac{1}{\alpha p + (1-\alpha)p_l}(1+0.5c) - 1.$$

As mentioned earlier, the \$2 loan is not the only way for an entrepreneur to fund his \$2 project. The alternative is to take out two separate \$1 loans. When choosing between these two methods of funding the project, the entrepreneur compares interest rates. That is, it is cheaper to fund the \$2 project using one \$2 loan as long as $p_j[2R - 2(1+r_2)] \geq p_j[2R - 2(1+r_1)]$, or $r_2 \leq r_1$, where $p_j \in \{p, p_l\}$. Observe that regardless of the entrepreneur's type, the constraint is the same.

Whether the \$2 loan works out to be cheaper or not depends on how the savings in transaction cost compare against the added risk premium. The \$2 loan has a lower transaction cost per dollar than the \$1 loan, but the \$2 loan attracts low ability borrowers, who have a lower probability of success. This means that the \$2 loans require an additional risk premium that is not present in the \$1 loan. The net impact of these two factors can go either way.

To organize our analysis, we break the following discussion into two parts. In the first part we consider the case where $r_2 \leq r_1$, and in the second part we consider the case where $r_2 > r_1$.

3.1 Competition and Efficiency

The interest rate that banks are willing to offer on the \$1 loans depends on exactly who the banks expect to demand such loans. One possibility is that the banks believe that there will not be any double borrowing. In this case, competition between banks generates a competitive interest rate of r_1 on each \$1 loan. To support this loan contract as part of equilibrium behavior, it is necessary that entrepreneurs prefer to not double borrow. As we explained earlier, entrepreneurs will not want to double borrow as long as $r_2 \leq r_1$, or

(2) $\qquad \alpha \geq \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$.

One can verify that Assumption A3 implies that this lower bound lies in the open interval $(0,1)$. This brings us to the following result.

Proposition 1. Let $\alpha \geq \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$. *In equilibrium, the banks offer* \$1 *loans at* r_1 *and* \$2 *loans at* r_2. *All entrepreneurs with access to the* \$2 *project invest in the project using a* \$2 *loan and all other entrepreneurs invest in the* \$1 *project. The resulting allocation is efficient.*

Proof. See appendix.

This result describes equilibrium behavior in the credit market when interest rates are declining in loan size. This occurs when a lower transaction cost per dollar dominates the added risk premium. The higher α is, the less significant is the presence of low ability entrepreneurs, which translates to a lower risk premium on the \$2 loan. In equilibrium, the investment choices made by the different entrepreneurs are efficient. Since the interest rates decline with loan size, there is no reason for entrepreneurs to pursue double borrowing. This minimizes the transactions costs associated with funding the entrepreneurs' investment projects. Aggregating the individual gains across the population of entrepreneurs generates a total expected gains from bank lending equal to

(3) $\qquad \lambda n\left[(\alpha p+(1-\alpha)p_l)2R-2-c-w\right]+(1-\lambda)n\left[pR-1-c-w\right]$.

To sustain this efficient outcome in the credit market, there needs to be sufficient number of high ability entrepreneurs. These entrepreneurs lower the impact that the riskier type of entrepreneur has on the interest rate r_2 for the \$2 loan. That is, as long as α is high enough, r_2 is less than r_1, and this implies that the entrepreneurs have no desire to seek out multiple loans. However, as α falls, r_2 rises, which makes double borrowing a more attractive proposition.

3.2 Competition and Inefficiency

The other possibility is one where if a bank offers \$1 loans at r_1, then this attracts double borrowing. This makes sense when $r_2 > r_1$. Entrepreneurs who plan to invest in the \$2 project now figure that it is cheaper to fund their project using two separate \$1 loans. Of course the banks anticipate this before the loans are actually issued. When entrepreneurs plan to double borrow, banks must recalculate the interest rate on the \$1 loan.

Consider a bank that offers \$1 loans. Denote this bank as *bank A*. Suppose that all $(1-\lambda)n$ entrepreneurs borrow from bank A. In addition, suppose that the same offer attracts all λn entrepreneurs, who double borrow. That is, these entrepreneurs take one loan from bank A and another \$1 loan from a different bank. Given this composition of different clients, bank A can afford to charge an interest rate where

$$(4) \qquad (1-\lambda)n[p(1+r_x)-(1+c)] + \alpha\lambda n[p(1+r_x)-(1+c)] + (1-\alpha)\lambda n[p_l(1+r_x)-(1+c)] = 0, \text{ or}$$

$$r_x = \frac{1}{\lambda[\alpha p+(1-\alpha)p_l]+(1-\lambda)p}(1+c) - 1.$$

One can verify that r_x exceeds r_1. This is because some of the borrowers are low ability. Thus, entrepreneurs who only take one loan are now forced to pay more than what they did in Proposition 1. However, it is this subsidy that attracts the double borrowers. That is, there is cross subsidization. While taking out two \$1 loans is relatively costly in terms of transaction costs, the $(1-\lambda)n$ entrepreneurs who only take one loan lower the average risk, which keeps the interest rate low.

In order for the λn entrepreneurs to double borrow, at least one other bank must also offer \$1 loans. However, the other banks cannot match bank A on his offer. The reason is that bank A is lending \$1 to all $(1-\lambda)n$ entrepreneurs, who only take one loan. These entrepreneurs are the source of the subsidy on the \$1 contract. If another bank tries to match bank A's offer, then the entrepreneurs become indifferent and the bank will attract only half of the $(1-\lambda)n$ entrepreneurs. Consequently, at r_x, the bank takes a loss.

Just to clarify, say that a bank does match bank A on his offer. Then all $(1-\lambda)n$ entrepreneurs are now indifferent, so the two banks split these clients evenly. The λn entrepreneurs double borrow, by taking out one loan from each of the two banks offering r_x. This implies that bank A and the deviating bank now each make

$$(5) \qquad \tfrac{1}{2}(1-\lambda)n[p(1+r_x)-(1+c)]+\alpha\lambda n[p(1+r_x)-(1+c)]+(1-\alpha)\lambda n[p_l(1+r_x)-(1+c)]<0.$$

When banks other than bank A offer \$1 loans, they must charge a rate above r_x. This implies that the banks will not attract any of the $(1-\lambda)n$ entrepreneurs who only take one loan. Suppose that all banks, except bank A, offer \$1 loans and that these banks attract the λn entrepreneurs, all of whom double borrow. In particular, each bank attracts $\frac{1}{m}$ of the λn entrepreneurs. In this case, a bank can afford to offer an interest rate where

$$(6) \qquad [\alpha p+(1-\alpha)p_l]\,(1+r_z)-1-c=0, \text{ or}$$

$$r_z=\tfrac{1}{\alpha p+(1-\alpha)p_l}(1+c)-1.$$ [7]

The λn entrepreneurs who double borrow take out one loan at r_x and the other loan at r_z. The alternative to double borrowing is of course to just take out one \$2 loan. Suppose a bank deviates and offers a \$2 loan. If this offer attracts an entrepreneur, it will attract both types, as we discussed earlier. Hence, offering the \$2 loan is worthwhile for the bank only if the bank charges an interest rate not less than r_2.[8] On the other hand,

[7] Note that this rate is similar in construction to r_2. While it has the same risk premium as r_2, r_z has a higher transaction cost per dollar of loan.

[8] This is true if the bank desires to earn non-negative profit on the \$2 contract. However, as we explain in the proof of Proposition 2, there can be rational for offering a loss inducing \$2 contract. In particular, offering such a contract can create (a short-lived) externality that creates profit on the \$1 contract.

an entrepreneur will only be interested in the \$2 loan if the rate is less than $\frac{1}{2}(r_x + r_z)$. Thus, to rule out the possibility of this kind of deviation we require that $\frac{1}{2}(r_x + r_z) < r_2$, or

$$(7) \qquad \lambda < \overline{\lambda} \equiv \frac{p-[\alpha p+(1-\alpha)p_l](1+c)}{p-[\alpha p+(1-\alpha)p_l]}.$$ [9]

Whether an entrepreneur prefers to double borrow or not depends on the fraction of the population that can scale up their production projects. If parameters dictate that $\overline{\lambda} > 0$, then entrepreneurs will prefer to take out multiple loans whenever λ is sufficiently low. This brings us to the following result.

Proposition 2. Let $\alpha < \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$. *Also, assume that* $p[R - \frac{1+0.5c}{\alpha p+(1-\alpha)p_l}] \geq w$.

I. If $\lambda < \frac{p-[\alpha p+(1-\alpha)p_l](1+c)}{p-[\alpha p+(1-\alpha)p_l]}$, *then in equilibrium one bank offers* \$1 *loans at* r_x *and all other banks offer* \$1 *loans at* r_z. *All entrepreneurs with* \$2 *projects invest in their projects by double borrowing and the remaining entrepreneurs invest in their* \$1 *projects. The allocation is inefficient due to double borrowing, resulting in an efficiency loss of* λnc.

II. Say that $\lambda \geq \frac{p-[\alpha p+(1-\alpha)p_l](1+c)}{p-[\alpha p+(1-\alpha)p_l]}$. *If* $r_x \geq r_2$, *then in equilibrium all banks offer* \$1 *loans and* \$2 *loans at* r_2, *but if* $r_x < r_2$ *then all banks offer* \$2 *loans at* r_2 *and one bank offers* \$1 *loans at* r_x. *All entrepreneurs with* \$2 *projects invest in their projects using* \$2 *loans and all other entrepreneurs invest in their* \$1 *projects. The allocation is efficient.*

Proof. See appendix.

When $r_2 > r_1$, we identify two possible equilibrium outcomes, depending on what λ is. When $\lambda < \overline{\lambda}$, competition between banks results in market interest rates that make it attractive for entrepreneurs to double borrow. Rather than take out \$2 loans, all entrepreneurs who own a \$2 project choose to fund the project using multiple loans from different banks. The entrepreneurs who double borrow are riskier on average than those

[9] The value $\overline{\lambda}$ is guaranteed to be less than one, but does not necessarily exceed zero.

who do not, and so, entrepreneurs who double borrow conceal their intentions by soliciting separate banks for the two different loans. The double borrowing by low ability entrepreneurs drives the interest rate up on the \$1 loan contract, but not to the point where entrepreneurs would prefer to borrow \$2 at r_2. While all investment decisions by the entrepreneurs are efficient, the double borrowing is inefficient due to excessive transaction costs.

Each entrepreneur who double borrows imposes a total transaction cost of $2c$ on the economy. We interpret this as a case where banks duplicate expensive screening and monitoring of the entrepreneurs, which is wasteful. It would be socially beneficial to bundle the two different \$1 loans into a single \$2 loan, all else equal. The reason this cannot be accomplished is that issuing the \$2 loan would reveal that the borrower is a higher expected risk than the average entrepreneur taking the \$1 contract at r_x. Consequently, an additional risk premium would be attached to the \$2 loan, and this would makes the loan undesirable to the entrepreneur.

The subsidy enjoyed by double borrowers with the \$1 contract originates from the $(1-\lambda)n$ entrepreneurs who only take out one loan. If an entrepreneur attempted to take two loans from the same bank, the bank would benefit by excluding this borrower from the loan portfolio, as he is relatively risky. This is why entrepreneurs who double borrow use different banks for their loans. This is different from what Bizer and Demarzo (1992) argue in their paper. In their model, the authors find that when an entrepreneur takes out multiple loans, the entrepreneur has no need to deal with more than one bank. Our result is different because of the cross subsidization, due to heterogeneity in the population of entrepreneurs.

We find that there is double borrowing in equilibrium when λ is sufficiently low. This means that a small portion of the population has an opportunity to scale up their projects. In contrast, when $\lambda \geq \overline{\lambda}$, the equilibrium allocation in the credit market is efficient. Entrepreneurs invest in the efficient projects and there is no double borrowing. While it is still the case that $r_2 > r_1$, the pooling rate on the \$1 contract, namely r_x, is high enough to make double borrowing unattractive. A high value of λ diminishes the importance of entrepreneurs who only invest in the \$1 project, which lowers the potential subsidy and thus, when comparing interest rates on \$1 loans and \$2 loans, it comes down to the transaction cost.

Since $\frac{1}{2}(r_x + r_z) \geq r_2$, the entrepreneurs prefer the \$2 loan contract over double borrowing at the interest rates

r_x and r_z. In equilibrium, the only entrepreneurs that borrow \$1 are the entrepreneurs who invest in the \$1 project. Hence, there is no longer any cross subsidization in the \$1 loan contract. Interestingly though, entrepreneurs still pay an equilibrium interest rate that is higher than the competitive rate, r_1. As described in the Proposition, the entrepreneurs pay either r_2 or r_x, whichever is lower. When banks charge r_2 on the \$1 contract, entrepreneurs with access to the \$2 project are indifferent between double borrowing and not. That is, the incentive compatibility constraint is binding. A side effect of this is that banks earn positive expected profit on every \$1 loan issued. Because of the binding constraint, Bertrand competition does not eliminate the banks' profits. If a bank tries to gain market share by lowering the interest rate on the \$1 loan, then the deviation attracts double borrowers.

In both of the equilibria described in Proposition 2, the entrepreneurs that only own the \$1 project choose to borrow. This behavior by itself, is efficient. However, the entrepreneurs do not pay r_1 on the loan. They either pay r_x or r_2, both of which exceed r_1. In the first case, the entrepreneurs subsidize double borrowers and in the second case, the entrepreneurs earn the banks positive profit. One implication of this is that the entrepreneurs may not want to borrow at all. That is, if the rate on the \$1 loan is too high, then the entrepreneur may prefer to earn the wage income w. We have ruled this out by assuming that $p[R - \frac{1+0.5c}{\alpha p+(1-\alpha)p_l}] \geq w$, which just means that the entrepreneur prefers to pay r_2 on a \$1 loan rather than earn w. In general though, the incentive for the entrepreneurs to participate in the credit market will not always hold. High interest rates for small loans can drive out the lower risk entrepreneurs seeking only one loan, and if this occurs, the credit market may fail.

4. Adverse Selection

In this section of the paper we study the possibility of adverse selection. We are interested whether competition between the banks can generate an outcome where the $(1-\lambda)n$ entrepreneurs choose to not borrow funds. From Proposition 1, we know that when parameters dictate that $r_2 \leq r_1$, the entrepreneurs can borrow \$1 at r_1. In this case, Assumption A1 guarantees that the entrepreneur's participation constraint holds. Hence, in this section, we turn our attention to the case where $r_2 > r_1$.

Consider a collection of player strategies where all $(1-\lambda)n$ entrepreneurs choose to earn wage income rather than borrow. Also, suppose that all banks offer \$2 loans at r_2 and none of the banks offer \$1 loans. The question here is whether we can support this as an equilibrium or not.

Say a bank deviates and offers \$1 loans. To attract the entrepreneurs who plan to invest in the \$1 project, the bank charges an interest rate where $p[R-(1+r)] \geq w$, or $r \leq r_w \equiv R - \frac{w}{p} - 1$. Assumption A1 implies that $r_w > r_1$. Thus, as long as $r \in [r_1, r_w]$, the deviation is potentially interesting to both the entrepreneurs and the bank.

This offer by itself is not at risk of attracting double borrowers. The reason is that if only one bank offers \$1 loans, then double borrowing is not feasible.[10] To discourage this kind of deviation what we require is an incentive for other banks to react to the deviation by offering \$1 loans themselves. Furthermore, we then need to show that this reaction renders the initial deviation unprofitable.

After the bank offers the \$1 loans, suppose that another bank reacts to the deviation by also offering \$1 loans. This now opens the possibility for double borrowing. To make double borrowing unprofitable for the bank making the original deviation, we focus on the case where at r_x, the $(1-\lambda)n$ entrepreneurs elect to not borrow. That is, assume that $p[R-(1+r_x)] < w$, which implies that $r_w < r_x$. This means that once the reacting bank offers loans and entrepreneurs begin to double borrow, the bank that is charging $r \in [r_1, r_w]$ now expects a loss on the \$1 contract.

The reacting bank does not attract any of the $(1-\lambda)n$ entrepreneurs. Rather, the bank only attracts double borrowers. This means that the reacting bank can afford to offer a rate as low as r_z, as calculated earlier. To make the reaction profitable for the bank, we require that entrepreneurs prefer to double borrow. This is true for any $r \in [r_1, r_w]$ as long as $\frac{1}{2}(r_w + r_z) < r_2$, or $p[R - \frac{1}{\alpha p + (1-\alpha)p_l}] < w$.

This brings us to the following result.

Proposition 3. Let $\alpha < \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$. *Also, assume that* $p[R - \frac{1+c}{\lambda[\alpha p+(1-\alpha)p_l]+(1-\lambda)p}] < w$. *If* $p[R -$

[10] The bank of course can limit the number of loans available to a single agent.

$\frac{1}{\alpha p+(1-\alpha)p_l}] < w$*, then in equilibrium all banks offer* $2 *loans at* r_2 *and no banks offer* $1 *loans. All entrepreneurs with* $2 *projects invest in the projects and no entrepreneurs invest in the* $1 *project. The equilibrium is inefficient due to the entrepreneurs who do not invest, resulting in an efficiency loss of* $(1-\lambda)n\,[pR-1-c-w]$.

Proof. See appendix.

This result confirms the possibility of market failure due to adverse selection. In equilibrium, banks only offer $2 loans. Entrepreneurs who own the $1 project are unable to obtain the funds necessary to invest in their projects. If a bank tries to offer small loans to these entrepreneurs, then this offer instigates other banks to offer additional $1 loans which in turn, leads to double borrowing in the credit market. Once entrepreneurs begin double borrowing, the bank that made the first offer finds that he is charging too low of a rate, and faces a loss. Anticipating all this, none of the banks choose to service this segment of the credit market. Thus, the small loan market fails.

In the previous section, we examined a lending strategy where the banks offered $1 loans at the same interest rate as the $2 loans. This precluded double borrowing due to the simple fact that entrepreneurs investing in the $2 project were indifferent. This can be an effective strategy for issuing small loans, as long as the entrepreneurs can afford to pay the rate. In Proposition 3 however, we are focused on a case where $\frac{1}{2}(r_w+r_z) < r_2$. Since $r_z > r_2$, $\frac{1}{2}(r_w+r_z) < r_2$ implies that it must be that $r_w < r_2$. This means that the entrepreneur with the $1 project is not willing to pay an interest rate of r_2 for a $1 loan.

In this case, the credit market for $1 loans fails because of what banks anticipate such a market would bring. Namely, once banks start offering small loans, double borrowing occurs, which means banks must charge an additional risk premium. However, the risk premium makes the loans expensive and consequently, the lower risk entrepreneurs who only want one loan figure that they can do better earning a wage.

5. Extensions

When entrepreneurs take out multiple loans, they do so from multiple different banks. As we explained earlier, the reason for this is because the entrepreneurs are relatively risky, and are covertly seeking the cross subsidy. Naturally, competition between the banks creates an incentive to sort out the different types of entrepreneur. In this regard, there are a few different extensions that one might try in order to capture this effort on the part of the banks. In this section of the paper, we explore three different extensions.

5.1 Exclusive Contracting

One rather obvious change is to introduce exclusive contracting. In this case, when a bank issues a loan we simply assume that the bank can stipulate that the entrepreneur is not allowed to take any other loans. The first implication of this is that all $(1-\lambda)n$ entrepreneurs immediately demand exclusivity on their \$1 loans. Competition between the banks to supply these loans results in an interest rate r_1. Given this contract, the possibility for cross subsidization is eliminated. That is, the cross subsidization where entrepreneurs with one \$1 loan subsidize the other λn entrepreneurs is eliminated. This is exactly why the entrepreneurs demand the exclusivity: to avoid having to pay the subsidy. Hence, the remaining λn entrepreneurs opt for \$2 loans at r_2.

Proposition 4. Under exclusive contracting, in equilibrium the banks offer \$1 *loans at* r_1 *and* \$2 *loans at* r_2*. All entrepreneurs with access to the* \$2 *project invest in the project using a* \$2 *loan, while all other entrepreneurs invest in the* \$1 *project. The resulting allocation is efficient.*

With exclusive contracting it no longer matters whether $r_2 > r_1$ or not. As we explained earlier, the incentive for an entrepreneur to double borrow occurs with the pooling contract, in which some entrepreneurs do not double borrow. Once the $(1-\lambda)n$ entrepreneurs secure their exclusive contracts, pooling is not viable and thus, the rational for double borrowing vanishes. Thus, the equilibrium outcome is always efficient. A potential problem with this solution is that exclusive contracting is often described as difficult or impossible to implement in practice. This is probably especially relevant given the intended application of the model; namely the small business loan and microfinance loan markets.

5.2 Communication

Another modification we can make to our model is to introduce communication between the banks. One might imagine that communication allows banks to better coordinate their lending behavior so as to avoid inefficient outcomes. A simple approach is to assume that after a bank issues a \$1 loan to an entrepreneur, the bank immediately informs all other banks that the entrepreneur received a \$1 loan. In order for this to be meaningful, we need to also rule out the possibility that the entrepreneur can take out two loans from different banks, simultaneously. To model this we can adopt a structure similar to the one used by Bizer and DeMarzo (1992), where the one period model has two separate phases. In the banking phase, an entrepreneur may request and obtain loans repeatedly, each from a new bank. During this phase, all banks are perfectly aware of the loans each entrepreneur takes out. The entrepreneur can keep requesting additional loans until satisfied. Once all entrepreneurs are satisfied, the banking phase ends and the game moves to the investment phase.

Working backwards, suppose that an entrepreneur approaches a bank requesting a \$1 loan. Furthermore, suppose that the bank observes that this entrepreneur has already been issued one \$1 loan. Clearly the entrepreneur is trying to double borrow and invest in the \$2 project. Since the incentive to double borrow is the same for both types of entrepreneur, the entrepreneur requesting his second \$1 loan can be high or low ability. This means that the bank can afford to offer the entrepreneur an interest rate as low as r_z.

Entrepreneurs who plan to double borrow do not have to pay r_z for their first loan, only their second loan. When an entrepreneur is taking out his first \$1 loan, the entrepreneur is not necessarily a double borrower. When an entrepreneurs asks for his first \$1 loan, the bank issuing the loan can afford to charge a rate of r_x. Entrepreneurs who plan to double borrow, obtain their first loan at r_x and their second loan at r_z. An entrepreneur prefers to double borrow if $\frac{1}{2}(r_x + r_z) < r_2$, or as we stated earlier, $\lambda < \overline{\lambda}$.

This is exactly what we found in Proposition 2. Thus, information sharing doesn't help. When banks share information about borrowing, we still find double borrowing in equilibrium. This is actually not that surprising. In Proposition 2, when banks issue \$1 loans at r_z they assume that any entrepreneur taking the loan is indeed double borrowing. In this sense, information sharing does not add any new information. Thus, we find that introducing communication between banks is redundant.

What is critical here is whether information sharing implies that exclusivity can be enforced as part of a loan contract. We have assumed that this is not true. If the bank offering the rate r_x finds out that his borrower took a second \$1 loan, then the after the bank updates his beliefs, the bank views an expected loss on the loan. In this sense, he now prefers that he hadn't issued it to begin with. However, it is not obvious that information sharing should imply that a bank can enforce exclusivity. In a microfinance market for example, if a bank finds out that a borrower has taken a second loan, it can be expensive or simply legally impossible for the bank to then request that the borrower return the loan.

5.3 Screening

One feature that is missing from our model is an ability of the bank to screen the heterogenous agents who plan to expand the size of their business projects. When an entrepreneur requests a larger loan size for his business, banks naturally will attempt to gauge how capable the entrepreneur is with managing the new project. There are a number of different ways we might incorporate this into our paper. One approach is to assume that within the set of λn entrepreneurs who have an opportunity to scale up their investment projects, the banks can discriminate to some degree between high and low ability entrepreneurs. In this part of the paper, we formalize this concept using a rather simple extension of the existing model.

Assume that if an entrepreneur selects a \$2 loan contract, then the bank can conduct a costless screening exercise in order to verify the entrepreneur's type before the loan is issued. To model this, assume that a subset of the $\alpha\lambda n$ high ability entrepreneurs are on a list, so to speak. When the bank runs the screening exercise and the entrepreneur is on this list, then the bank observes that the entrepreneur is high ability. Assume that there are $\phi\alpha\lambda n$ high ability entrepreneurs on the list, where ϕ is a parameter and $\phi \in [0, 1]$.[11] To keep things simple,

[11] We also assume that all λn agents know ex ante whether they are on the list or not.

assume that all banks observe the same list.[12] Consequently, if $\phi = 1$, then banks effectively have perfect information, but if $\phi = 0$, then banks are unable to verify the type of any entrepreneur, as in Section 3.

Now consider competition between the different banks. For entrepreneurs on the list, banks offer a competitive interest rate of $r_\phi = \frac{2+c}{2p} - 1$. This offer is *conditional* on the entrepreneur being high ability. Note that $r_\phi < r_1$. Thus, entrepreneurs who are identified as high ability never want to double borrow. This leaves the $(1-\phi\alpha)\lambda n$ entrepreneurs with access to the \$2 project, who do not pass the screening test. If entrepreneurs in this group seek an *unconditional* \$2 loan, a bank can afford to charge an interest rate where

$$\text{(8)} \qquad \left[\tfrac{(1-\phi)\alpha}{(1-\phi\alpha)}p + \tfrac{(1-\alpha)}{(1-\phi\alpha)}p_l\right] 2(1+r) - 2 - c = 0, \text{ or}$$

$$r_2(\phi) = \tfrac{1-\phi\alpha}{(1-\phi)\alpha p + (1-\alpha)p_l}(1 + \tfrac{1}{2}c) - 1.$$

Note that if $\phi = 0$, then $r_2(\phi) = r_2$, as in Section 3. One possibility is that $r_2(\phi) \leq r_1$. When this holds, the entrepreneurs prefer the \$2 loan over double borrowing. The more interesting case is when $r_2(\phi) > r_1$. Assume that $r_2(\phi) > r_1$. Suppose that banks do not issue any \$2 loans unless an entrepreneur passes the screening test. Let one bank, denoted as bank A, offer \$1 loans and assume this offer attracts all $(1-\lambda)n$ entrepreneurs who take one loan, and the $(1-\phi\alpha)\lambda n$ entrepreneurs who plan to double borrow. Then bank A can afford to charge an interest rate as low as

$$\text{(9)} \qquad r_x(\phi) = \tfrac{1-\phi\alpha\lambda}{\lambda[(1-\phi)\alpha p + (1-\alpha)p_l] + (1-\lambda)p}(1+c) - 1.$$

Furthermore, suppose all other banks in the credit market offer \$1 loans at a rate that only attracts the $(1-\phi\alpha)\lambda n$ entrepreneurs who plan to double borrow. These banks can afford an interest rate of

$$\text{(10)} \qquad r_z(\phi) = \tfrac{1-\phi\alpha}{(1-\phi)\alpha p + (1-\alpha)p_l}(1+c) - 1.$$

[12] In practice, the ability of multiple banks to identify the same group of low risk clients could have to do with legislation governing credit transactions. For example, reforms that make it easier to identify and/or collect borrower assets could imply that certain types of businesses are more creditworthy.

Given the two different interest rates $r_x(\phi)$ and $r_z(\phi)$, an entrepreneur will want to double borrow only if $\frac{1}{2}(r_x(\phi) + r_z(\phi)) < r_2(\phi)$, or

$$(11) \qquad \lambda \leq \overline{\lambda}_s \equiv \frac{p(1-\phi\alpha)-[(1-\phi)\alpha p+(1-\alpha)p_l](1+c)}{p(1-\phi\alpha)-[(1-\phi)\alpha p+(1-\alpha)p_l](1+\phi\alpha c)}.$$

This gives us the following result.

Proposition 5. Let (i.) $\alpha < \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$*, or (ii.)* $\alpha \geq \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$ *and* $\phi > \frac{[\alpha p+(1-\alpha)p_l](1+c)-p(1+0.5c)}{\alpha p0.5c}$*. Also let* $p[R - \frac{1+0.5c}{p_l}] \geq w$*. If* $\lambda \leq \frac{p(1-\phi\alpha)-[(1-\phi)\alpha p+(1-\alpha)p_l](1+c)}{p(1-\phi\alpha)-[(1-\phi)\alpha p+(1-\alpha)p_l](1+\phi\alpha c)}$*, then in equilibrium one bank offers \$1 loans at* $r_x(\phi)$ *and all other banks offer \$1 loans at* $r_z(\phi)$*. All entrepreneurs with \$2 projects invest in the project by double borrowing and the remaining entrepreneurs invest in their \$1 projects. The allocation is inefficient due to the double borrowing, resulting in an efficiency loss of* $\lambda n(1-\phi\alpha)c$*.*

Proof. See appendix.

This result applies for the case where $r_2(\phi) > r_1$. This inequality can hold for two different reasons. If the fraction of high ability entrepreneurs is low enough, then this by itself implies that $r_2(\phi) > r_1$. This is identical to what we derived in Section 3. If on the other hand, the fraction of high ability entrepreneurs is not low enough, then $r_2(\phi) > r_1$ as long as the screening technology used by the banks is good enough. At $\phi = 1$, $r_2(1) > r_1$ for any feasible value of α.[13] The fact that double borrowing can be supported in equilibrium for any feasible value of α is a change from our earlier findings. The higher ϕ is, the more effective banks are at identifying low risk entrepreneurs and hence, the lower is the average quality of borrower seeking funds at the rate $r_2(\phi)$. This puts upward pressure on the interest rate $r_2(\phi)$. That is, the screening effort by the banks leaves behind a relatively riskier group of entrepreneurs who find that it is cheaper to take out several small loans rather than pay what banks would charge them they asked for a single \$2 loan.

[13] This is due to Assumption A3.

However, there is an upside to bank screening. While screening worsens the quality of the pool of entrepreneurs who go on to seek multiple loans, it also implies that less entrepreneurs do so. When banks screen, it is only those entrepreneurs who do not pass the screening test that choose to double borrow. The entrepreneurs who pass the test go on to secure \$2 loans at a low interest rate. Comparing Propositions 2 and 5, one can indeed confirm that the efficiency loss is less when banks screen, i.e., $\phi > 0$. Thus, while screening may make double borrowing more likely, it also makes it less relevant.

6. Conclusion

In this paper we have argued that multiple borrowing can occur when entrepreneurs seek out cheaper ways to fund their relatively risky business expansions. By concealing their intentions from the banks, the entrepreneurs obtain a lower interest rate by taking out several small loans rather than one larger loan. While this generates high transaction costs, there is a benefit due to cross-subsidization in the loan contract. The subsidy originates from the relatively safer entrepreneurs who do not plan to expand their business projects. There is competition between the banks to attract the entrepreneurs who do not multiple borrow. The bank that wins the majority of these clients is then able to charge the lowest interest rate. The result is that banks offer different interest rates on small loans in equilibrium. One bank services a mix of different clients at a relatively low interest rate, while the other banks offer the same type of loan at a higher interest rate.

When multiple borrowing occurs in equilibrium, the banks do not offer large sized loans. The only recourse for funding a business expansion is in fact to multiple borrow, even for those entrepreneur's who have relatively safe projects planned. We also find that in other equilibrium outcomes, where entrepreneurs do not multiple borrow, the prospect that it can occur has important implications. One is that banks offer a high interest rate on small loans, in order to dissuade entrepreneurs from trying to multiple borrow. In this case, banks earn positive profit in equilibrium. The other is that if the interest rate for small loans is driven high enough, the safer segment of the borrowing population elects to exit the credit market and the small loan market fails. In this event, entrepreneurs are unable to access the funding necessary to make efficient investments.

The results of our study suggest that multiple borrowing is associated with riskier types of firms, on average. In order to sustain multiple borrowing in equilibrium, α must be sufficiently low, which implies a high fraction of the risky type of entrepreneur. There is empirical evidence that points in this direction. For example, Harhoff and Korting (1998) and Farinha and Santos (2002) find that multiple borrowing tends to be associated with weaker types of firms. Our results also predict that interest rates in the market for small loans can remain high, even with price competition between banks. Either multiple borrowing drives the rate up, or the prospect of multiple borrowing forces banks to keep the rate high. These results may help to explain some of the evidence of high interest rates documented by Rosenberg et al. (2013), in their survey of global microfinance markets.[14]

We also make an argument why in a competitive credit market, banks may elect to not offer larger size business loans. The obstacle here is that the larger projects involve a risk premium that the entrepreneurs prefer not to pay. Instead, they tolerate high transactions costs under multiple lending relationships as a means of enjoying the lower interest rate on the pooling contract for small loans. While this works to the entrepreneur's advantage, it is inefficient. This point resonates with some of the policy discussion about whether microfinance institutions are innovative enough in terms of offering new contracts and attracting new segments of the credit market.[15] For example, in an empirical paper, Field et al. (2013) find evidence that lengthening the amount of time between disbursement of a loan and when repayment must begin can have positive and significant effects on business investment and long-run profit for entrepreneurs.

As mentioned above, when multiple borrowing occurs, we find that banks offer different interest rates for small loans. Clearly this implies that the entrepreneur who multiple borrows would prefer to get more of his funds from the bank offering the lower rate. Of course this is not feasible in our model given the fixed loan sizes, and furthermore, it raises questions about whether such demand would backfire by revealing type. However, our initial findings may offer a good starting point for trying to better understand the empirical observation raised

[14] Interestingly, Rosenberg et al. (2013) attempt to measure the different components of observed interest rates. In this regard, they find that operating expense explains roughly the same amount as loan loss provisions, profit and financial expense, combined.

[15] For example, see Karlan and Mullainathan (2006).

by Guiso and Minetti (2010); namely, that firms often borrow a different amount of funds from their different banks.

Appendix

Proof of Proposition 1. This result applies when $r_2 \leq r_1$. Consider the incentives facing the entrepreneurs. One choice is whether to scale up or not. When choosing between a \$2 loan at r_2 and one \$1 loan at r_1, the low ability entrepreneur prefers the \$2 loan as long as $p_l[2R - 2(1+r_2)] > p[R - (1+r_1)]$, or $p_l 2R > pR + 1$, which holds under Assumption A2. Given that low ability entrepreneurs prefer to scale-up, high ability entrepreneurs do too. Since $r_2 \leq r_1$, an entrepreneur clearly has no reason to take out two \$1 loans at r_1. Lastly, given a choice between borrowing \$1 at r_1 or earning a wage, Assumption A1 implies that the entrepreneur prefers to borrow.

With \$2 loans, any deviation by a bank who offers $r < r_2$ attracts λn entrepreneurs but generates negative expected profit. If a bank deviates and offers a \$1 loan at $r < r_1$ then this attracts the $(1-\lambda)n$ entrepreneurs. In this case, there are two possibilities. If $\frac{1}{2}(r + r_1) \geq r_2$, then the deviation only attracts the $(1-\lambda)n$ entrepreneurs and bank earns negative expected profit. On the other hand, if $\frac{1}{2}(r + r_1) < r_2$, then the offer also attracts λn entrepreneurs who double borrow. Namely, these entrepreneurs takes one \$1 loan at r, and one \$1 loan at r_1 from a different bank. However, the presence of low ability entrepreneurs among the double borrowers implies additional risk, which means that the bank offering r earns negative expected profit. QED

Proof of Proposition 2. Under the conditions given, $r_2 > r_1$. The result describes two equilibria, depending on whether $\frac{1}{2}(r_x + r_z) < r_2$ or not.

I. Suppose that $\frac{1}{2}(r_x + r_z) < r_2$. Denote the bank offering the rate r_x as bank A. If a bank other than bank A deviates and offers \$1 loans at $r < r_x$, then this attracts all n entrepreneurs in the economy, but earns the bank negative expected profit. If instead the bank offers $r = r_x$, then the bank only attracts half of the $(1-\lambda)n$ entrepreneurs. The bank also attracts all λn entrepreneurs, who double borrow. That is, these entrepreneurs take one loan from the bank and one loan from bank A. However, at r_x, the bank now expects a loss. If the deviation is at $r > r_x$, then the bank only attracts the λn entrepreneurs, which results in a loss for all $r < r_z$. Clearly bank A has no incentive to offer \$1 loans at $r < r_x$. If bank A offers \$1 loans at $r > r_x$ (instead of

r_x), then a different bank will react by offering the rate r_x itself. This reaction generates zero expected profit for the bank that is reacting, but as we explained in Section 2, banks always elect to lend rather than not lend when indifferent. This means that bank A will not attract any of the $(1-\lambda)n$ entrepreneurs at r.

If bank A deviates and offers a \$2 loan, then this only attracts an entrepreneur when $r < \frac{1}{2}(r_x + r_z)$. Say that $r = \frac{1}{2}(r_x + r_z) - \varepsilon$, for some $\varepsilon > 0$. This offer attracts both high and low ability entrepreneurs. Since $r < r_2$, the bank earns negative expected profit on the contract. However, when the bank issues this loan it affects the composition of entrepreneurs taking bank A's \$1 loans. In particular, one less entrepreneur double borrows. In general, say that m entrepreneurs stop double borrowing in the \$1 loan market. Given that each of these entrepreneurs is equally likely to stop, bank A now earns an aggregate expected profit from his \$1 loan contracts equal to

$$(12) \qquad ((1-\lambda)n + \alpha(\lambda n - m))[p(1+r_x) - (1+c)] + ((1-\alpha)(\lambda n - m)\,[p_l(1+r_x) - (1+c)].$$

The derivative of this profit with respect to m is positive. This implies that while bank A earns negative expected profit on the \$2 contract itself, the deviation serves to generate positive expected profit on each of the bank's existing \$1 loan contracts. However, after the deviation, the other banks in the credit market have an incentive to react, in the fashion described by Riley (1979). In this case, a different bank can undercut bank A on the \$1 contract. Consequently, the bank that made the original deviation earns negative expected profit on his loan portfolio due the loss inducing \$2 loan contract that cannot be withdrawn.

Lastly, we must verify the entrepreneurs' incentives. An entrepreneur prefers to borrow \$1 over earning a wage when $p[R-(1+r_x)] \geq w$. This holds because $r_x < r_2$ and we have assumed the entrepreneur prefers to borrow \$1 at r_2. A low ability entrepreneur prefers to double borrow rather than only take one \$1 loan as long as $p_l[2R - (2 + r_x + r_z)] \geq p[R - (1 + r_x)]$, or

$$(13) \qquad p_l 2R + (p - p_l)\tfrac{1+c}{\lambda[\alpha p + (1-\alpha)p_l] + (1-\lambda)p} \geq pR + p_l \tfrac{1+c}{\alpha p + (1-\alpha)p_l}.$$

For sufficiency, we can evaluate this inequality at $\lambda = 0$ and $\alpha = 0$, in which case we have $p_l 2R \geq pR + \frac{p_l}{p}(1+c)$. This holds under Assumption A2. Finally, since the low ability entrepreneur prefers to scale-up, so does the high ability entrepreneur.

II. Suppose that $\frac{1}{2}(r_x + r_z) \geq r_2$. First consider the case where $r_x \geq r_2$. If a bank deviates and offers a \$2 loan at a rate below r_2 all λn entrepreneurs accept the contract and the bank earns a negative expected profit. If a bank offers a \$1 contract below r_2, then the offer attracts all n entrepreneurs in the economy, where the λn entrepreneurs now double borrow. However, since $r_x \geq r_2$, this deviation yields the bank negative expected profit. Second, consider the case where $r_x < r_2$. Suppose a bank not offering the \$1 contract deviates and offers a \$1 contract. At $r < r_x$ this offer attracts all n entrepreneurs, λn of whom now choose to double borrow. This offer is unprofitable. At $r = r_x$ the offer attracts half of the $(1-\lambda)n$ entrepreneurs and all λn entrepreneurs, where all λn entrepreneurs now double borrow. Thus, the deviation is unprofitable. At $r > r_x$, the \$1 offer will not attract any of the $(1-\lambda)n$ entrepreneurs, in which case the bank must charge at least r_z, and thus, no entrepreneurs are interested. Now, alternatively, suppose that the bank offering the \$1 loans deviates and offers $r < r_x$. This continues to attract all $(1-\lambda)n$ entrepreneurs, but will not earn the bank higher profit. If the bank deviates and offers $r > r_x$, then a different bank can react and offer r_x itself. Consequently, the bank making the original deviation is worse off, as r_x was originally profitable. Finally, suppose that one of the banks deviates and offers a \$2 loan at $r < r_2$. This attracts all λn entrepreneurs, but earns the bank negative expected profit.

As far as the entrepreneurs' incentives, note that all rates offered in equilibrium do not exceed r_2. Thus, by assumption the entrepreneur prefers to borrow \$1 rather than earn w. A low ability entrepreneur prefers to scale up rather than not, if $p_l[2R - 2(1+r_2)] \geq p[R - (1+r_x)]$, or

$$(14) \qquad p_l 2R + p\frac{1+c}{\lambda[\alpha p + (1-\alpha)p_l] + (1-\lambda)p} \geq pR + p_l\frac{2+c}{\alpha p + (1-\alpha)p_l}.$$

For sufficiency, we can evaluate this inequality at $\lambda = 0$ and $\alpha = 0$, in which case we have $p_l 2R \geq pR + 1$. This holds under Assumption A2. Finally, since the low ability entrepreneur prefers to scale-up, so does the high ability entrepreneur. QED

Proof of Proposition 3. The conditions imply that $r_2 > r_1$. Suppose a bank deviates and offers $r \leq r_w$ on a \$1 loan. Then other banks will react to the deviation and offer \$1 loans at a rate r_z. entrepreneurs now double borrow and the reacting banks makes zero expected profit on their \$1 loans. Due to the double borrowing, the bank offering $r \leq r_w$ attracts all n entrepreneurs in the economy and earns negative expected profit because $r_w < r_x$. Obviously there is no reason for a bank to deviate and offer \$2 loans at a rate other than r_2.

entrepreneurs without access to a \$2 project have no other choice but to earn wage income. entrepreneurs with access to the \$2 project prefer to borrow \$2 at r_2 rather than earn w if $p_l[2R - 2(1 + r_2)] \geq w$, or $p_l 2R \geq \frac{p_l}{\alpha p+(1-\alpha)p_l}(2 + c) + w$. Assumptions A1 and A2 imply that this holds. Given that low quality entrepreneurs prefer to borrow, it follows that high ability entrepreneurs do as well. QED

Proof of Proposition 5. First we identify when $r_2(\phi) > r_1$. $r_2(\phi)$ is lowest at $\phi = 0$, and $r_2(0) > r_1$ if $\alpha < \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$. On the other hand, say $\alpha \geq \frac{(p-p_l)(1+c)-p0.5c}{(p-p_l)(1+c)}$. Then $r_2(\phi) > r_1$ if ϕ is high enough. At $\phi = 1$, $r_2(1) > r_1$ due to A3. To calculate the cutoff for ϕ we can solve $r_2(\phi) > r_1$ for ϕ, which yields $\phi > \frac{[\alpha p+(1-\alpha)p_l](1+c)-p(1+0.5c)}{\alpha p0.5c}$.

The inequality $\lambda \leq \frac{p(1-\phi\alpha)-[(1-\phi)\alpha p+(1-\alpha)p_l](1+c)}{p(1-\phi\alpha)-[(1-\phi)\alpha p+(1-\alpha)p_l](1+\phi\alpha c)}$ means that $\frac{1}{2}(r_x(\phi) + r_z(\phi)) < r_2(\phi)$. Suppose that the bank offering the rate $r_x(\phi)$ deviates and offers an unconditional \$2 loan. This offer only attracts an entrepreneur if $r < \frac{1}{2}(r_x(\phi) + r_z(\phi))$. Let $r = \frac{1}{2}(r_x(\phi) + r_z(\phi)) - \varepsilon$, for some $\varepsilon > 0$. The bank's expected profit on this single contract is

$$\text{(15)} \qquad \left[\frac{(1-\phi)\alpha}{(1-\phi\alpha)}p + \frac{(1-\alpha)}{(1-\phi\alpha)}p_l\right] 2(1 + \tfrac{1}{2}(r_x(\phi) + r_z(\phi)) - \varepsilon) - 2 - c < 0.$$

As in the proof of Proposition 2, the \$2 contract itself yields negative expected profit, but the contract generates positive expected profit on the \$1 contracts. However, other banks will react to the deviation and consequently, Bertrand competition leaves the deviating bank with a negative payoff due to the \$2 loan. If a bank other than the one offering $r_x(\phi)$ deviates and offers an unconditional \$2 loan, then any offer an entrepreneurs accepts will earn the bank negative expected profit.

If the bank offering $r_x(\phi)$ chooses to deviate and offer $r > r_x(\phi)$, then a different bank will react and offer $r_x(\phi)$, thus attracting all $(1-\lambda)n$ entrepreneurs. If a bank other than the one offering $r_x(\phi)$ deviates and offers a \$1 loan at $r < r_x(\phi)$, he will attract all n entrepreneurs in the economy, but due to the double borrowing, will take a loss. At $r = r_x(\phi)$ the bank only attracts half of the $(1-\lambda)n$ entrepreneurs, which implies a loss, and at $r > r_x(\phi)$ the bank does not attract any of the $(1-\lambda)n$ entrepreneurs.

Finally, consider incentives facing the entrepreneurs. Since $p[R - \frac{1+0.5c}{p_l}] \geq w$, an entrepreneur prefers to borrow \$1 rather than earn a wage for as long as $r \leq r_2(\phi)$ for any value of ϕ. A low ability entrepreneur prefers to scale up rather than not, as long as $p_l[2R - (2 + r_x(\phi) + r_z(\phi))] \geq p[R - (1 + r_x(\phi))]$, or $p_l 2R + (p - p_l)(1 + r_x(\phi)) \geq pR + p_l(1 + r_z(\phi))$. Note that, $r_x(\phi)$ is increasing in λ. So, for sufficiency, we can use $\lambda = 0$, which reduces the inequality to $p_l 2R + (p - p_l)\frac{1+c}{p} \geq pR + p_l(1 + r_z(\phi))$. Furthermore, $r_z(\phi)$ is increasing in ϕ. Hence, for sufficiency, we can use $\phi = 1$, which further reduces the inequality to $p_l 2R + 1 + c \geq pR + \frac{1}{p_l}(1 + c) + \frac{p_l}{p}(1 + c)$. This holds due to Assumption A2. Finally, since low ability entrepreneurs prefer to scale up, so do high ability entrepreneurs. QED

References

Andersen, T.B. and N. Malchow-Moller (2006) Strategic interaction in undeveloped credit markets, Journal of Development Economics, 80, 275-298.

Berger, A.N. and R.G. Udell (2002) Small business credit availability and relationship lending: The importance of bank organizational structure, Economic Journal, 112, F32-F53.

Bisin, A. and D. Guaitoli (2004) Moral hazard and nonexclusive contracts, RAND Journal of Economics, 35, 306-328.

Bizer, D.S. and P.M. DeMarzo (1992) Sequential banking, Journal of Political Economy, 100, 41-61.

Bolton, P. and D.S. Scharfstein (1996) Optimal debt structure and the number of creditors, Journal of Political Economy, 104, 1-25.

Broecker, T. (1990) Credit-worthiness tests and interbank competition, Econometrica, 58, 429-452.

Carletti, E. (2004) The structure of relationship lending, endogenous monitoring and loan rates, Journal of Financial Intermediation, 13, 58-86.

Carletti, E., V. Cerasi and S. Daltung (2007) Multiple-bank lending: diversification and free-riding in monitoring, Journal of Financial Intermediation, 16, 425-451.

Chen, G., S. Rasmussen and X. Reille (2010) Growth and vulnerabilities in microfinance, Focus Note, No. 61, CGAP.

Chua, R.T. and E.R. Tiongson (2012) Multiple borrowing in the Philippines: A pilot area study based on branch client data, Philippine Microfinance Discussion Papers, No. 2012-02.

Cull, R., A. Demirguc-Kunt and J. Morduch (2009) Microfinance meets the market, Journal of Economic Perspectives, 23, 167-192.

Detragiache, E., P. Garella and L. Guiso (2000) Multiple versus single bank relationships: theory and evidence, Journal of Finance, 55, 1133-1161.

Dewatripont, M. and E. Maskin (1995) Credit and efficiency in centralized and decentralized economies, Review of Economic Studies, 62, 541-555.

Diamond, D.W. (1984) Financial institutions and delegated monitoring, Review of Economic Studies, 51, 393-414.

Direr, A. (2008) Multiple equilibria in markets with screening, Journal of Money, Credit and Banking, 40, 791-798.

Farinha, L.A. and J.A.C. Santos (2002) Switching from single to multiple bank lending relationships: determinants and implications, Journal of Financial Intermediation, 11, 124-151.

Field, E., R. Pande, J. Papp and N. Rigol (2013) Does the classical microfinance model discourage entrepreneurship among the poor? American Economic Review, 103, 2196-2226.

Frisancho, V. (2012) Signaling creditworthiness in Peruvian microfinance markets: the role of information sharing, IDB working paper series, No. IDB-WP-347.

Gine, X. (2011) Access to capital in rural Thailand: An estimated model of formal vs. informal credit, Journal of Development Economics, 96, 16-29.

Guiso, L and R. Minetti (2010) The structure of multiple credit relationships: Evidence from U.S. firms, Journal of Money, Credit and Banking, 42, 1037-1071.

Harhoff, D. and T. Korting (1998) Lending relationships in Germany: empirical evidence from survey data, Journal of Banking and Finance, 22, 1317-1353.

Hubert, F. and D. Schafer (2002) Coordination failure with multiple-source lending, the cost of protection against a powerful lender, Journal of Institutional and Theoretical Economics, 158, 256-275.

Jain, S. (1999) Symbiosis vs. crowding out: The interactions of formal and informal credit markets in developing countries, Journal of Development Economics, 59, 419-444.

Karlan, D. and S. Mullainathan (2006) Is microfinance too rigid? Eye on Microfinance, Issue 2, August.

Khan, C.M. and D. Mookherjee (1998) Competition and incentives with non-exclusive contracts, RAND Journal of Economics, 29, 443-465.

Khandker, S.R. and H.A. Samad (2014) Dynamic effects of microcredit in Bangladesh, World Bank Policy Research WP 6821.

Livshits, I., J. MacGee and M. Tertilt (2011) Costly contracts and consumer credit, NBER Working Paper, No. 17448.

Marquez, R. (2002) Competition, adverse selection, and information dispersion in the banking industry, Review of Financial Studies, 15, 901-926.

McIntosh, C., A. de Janvry and E. Sadoulet (2005) How rising competition among microfinance institutions affects incumbent lenders, Economic Journal, 115, 987-1004.

McIntosh, C. and B. Wydick (2005) Competition and Microfinance, Journal of Development Economics, 78, 271-298.

Ongena, S. and D.C. Smith (2000) What determines the number of bank relationships? Cross-country evidence, Journal of Financial Intermediation, 9, 26-56.

Parour, C.A. and U. Rajan (2001) Competition in loan contracts, American Economic Review, 91, 1311-1328.

Peterson, M.A. and R.G. Rajan (1994) The benefits of lending relationships: Evidence from small business data, Journal of Finance, 49, 3-37.

Rajan, R. (1992) Insiders and outsiders: The choice between informed and arms-length debt, Journal of Finance, 47, 1367-1400.

Riley, J.G. (1979) Informational equilibrium, Econometrica, 47, 331-359.

Riordan, M. (1993) Competition and bank performance: A theoretical perspective, in C. Mayer and X. Vives (eds.), Capital Markets and Financial Intermediation, Cambridge University Press, Cambridge

Rosenberg, R., S. Gaul, W. Ford and O. Tomilova (2013) Microcredit interest rates and their determinants 2004-2011, Access to Finance, CGAP, No. &, June.

Schicks, J. and R. Rosenberg (2011) Too much microcredit? A survey of the evidence on over-indebtedness, Occasional Paper, 19, CGAP.

Sharpe, S. (1990) Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships, Journal of Finance, 45, 1069-1087.

Von Thadden, E. (2004) Asymmetric information, bank lending and implicit contracts: the winner's curse, Finance Research Letters, 1, 11-23.