A Case for Bundling Public Goods Contributions?*

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Abstract

We extend the model of voluntary contributions to multiple public goods by allowing for bundling of the public goods. Specifically, we study the case where agents contribute into a common pool which is then allocated towards the financing of two pure public goods. We explore the welfare implications of allowing for such bundling vis-a-vis a separate contributions scheme. We show that when agents have homogeneous preferences, they cannot be made better off with a bundling scheme. On the contrary, in the generic case when agents are heterogenous in their incomes and preferences, bundling may increase joint welfare compared to a separate contribution scheme, in particular for higher income inequality among the agents. It is interesting to note that the welfare improvement occurs despite a *decrease* in total contributions. Our findings have implications for the design of charitable institutions and international aid agencies.

Keywords: private provision, public goods, bundling

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1 Introduction

This paper analyzes a simple question: in a situation where agents contribute voluntarily to a single private good and multiple pure public goods, can there be a welfare improvement if contributions to the public goods are bundled? More precisely, we ask whether agents can be better off if, rather than contributing to the multiple public goods separately, they contribute a sum total which is divided among the public goods according to a pre-announced sharing rule. We find that when agents have identical preferences they cannot be made better off by such a joint contribution scheme regardless of the resource heterogeneity between them. On the contrary, in the non-degenerate case when there is heterogeneity among the agents in their preferences towards the public goods then there are plausible scenarios where a joint contribution scheme might increase total welfare.

There exist numerous real world examples of multiple public goods which are voluntarily supplied at the same time. National governments simultaneously contribute to many international public goods, many individuals contribute to several charitable causes, local sports clubs, etc. Our model also has implications for the design of organizations based on voluntary giving. For example the United Way of America, a leading charity organization, collected 4.4 billion dollars in the year 2002-2003 from various donors. These funds were then allocated for various projects aimed at "... helping children and youth succeed, strengthening and supporting families, promoting self-sufficiency, building vital and safe neighborhoods, and supporting vulnerable and aging populations."[1] A pertinent question to our analysis would be, should the donors be allowed to make project-specific contributions instead? As another example, the alumni of universities can make contingent donations to specific areas, say for the college football team or to a particular academic unit.

Similar issues arise in relation to disaster aid, for instance the 2005 tsunami disaster in Asia. There are broadly speaking three possible uses for aid - rescue, rehabilitation and relief, all of which are important but not always "optimally" provided. Rehabilitation and relief have strong public good aspects and are financed to a large extent (especially in poorer countries) with voluntary donations that are often mediated by international non-profit organizations such as the UN or the Red Cross. A further, albeit less immediate application of our results could arise in the area of campaign contributions. Voters or firms could contribute money to parties or politicians who then allocate it towards supporting various issues of public good character within a given industry. Alternatively, as required by the recent McCain-Feingold Bill, one could put a ban on the soft money received by parties while allowing direct contributions to candidates. Our paper could be used to shed some light on the welfare implications of these alternative schemes.

To analyze the problem outlined above we adopt the simplest possible theoretical structure focusing on expositional clarity and intuition rather than analytical generality. Specifically, we consider an economy with two agents and two public goods. The agents have certain resource (wealth) endowments which they can use for contributing to the public goods or private consumption. We first study the case of separate provision, where agents simultaneously decide to contribute separately to the two public goods. Next we analyze the equilibrium in a joint contribution game in which the agents make a total contribution towards the two public goods given an allocation rule in which the total contribution will be divided between the two public goods. We show that the welfare results for the two schemes depend crucially on the two sources of heterogeneity in our model. With heterogeneous preferences, for high wealth inequality among the agents, the joint contribution scheme performs better in terms of total welfare than the separate contribution scheme. Notice, however, that this does not imply that

[1] See http://national.unitedway.org/aboutuw/ciagenda.cfm

joint provision Pareto dominates separate provision.

In view of the free riding problems associated with public good provision, this paper proposes a simple mechanism that can, under wealth and preference heterogeneity, raise social welfare by requiring total voluntary contributions to be distributed towards the two public goods in a way closer to what would be the first best allocation. Under separate provision, and especially when wealth inequality is high, the richer agent's preferences determine to a large extent the provision level of the two public goods (simply because he is the one contributing the most towards them). This may be socially suboptimal in a utilitarian framework where both agents have the same weight in the social welfare function.

There are two basic channels through which a joint contributions scheme may lead to higher social surplus when agents' preferences are heterogeneous. First, joint provision can elicit contribution from some agents who would not contribute to a given public good under a separate provision scheme. This creates positive externalities for all other agents and can help increase joint welfare. Second, if the ex-ante sharing rule is chosen to maximize the (ex-post) social surplus resulting in equilibrium under the joint scheme, then it plays a "redistribution" role that can offset some of the social inefficiency in the separate scheme caused by the differences between the players' valuations and the differences in their abilities to contribute.

We find that there exist two countervailing forces which are at work when we switch from a separate to a joint contribution scheme. On the one hand, the mix of public goods that is provided in the joint scheme is social welfare enhancing compared to the separate case[2]. On the other hand, the total contribution to public goods (weakly) decreases because bundling constraints and distorts the way agents distribute their contributions to the two public goods which has a welfare reducing effect. A social welfare improvement with bundled contributions occurs when the first effect dominates the second effect. We show that this happens when the degree of wealth and preference heterogeneity among the agents is high.

The result that in a second best environment a policy such as bundling may increase social welfare despite reducing total contributions to the public goods is interesting in itself. It enables us to make a cautionary point about the "standard" presumption that, since public goods are normally underprovided due to free riding, a raise in total contributions must automatically increase welfare. While this intuition certainly holds in the commonly used in the literature single public good model, we show how it can easily fail when there are more public goods which people value differentially.

The majority of the existing literature on privately provided public goods, initiated by the seminal contributions of Bergstrom, Blume and Varian(1986), Cornes and Sandler (1985) and Andreoni (1988) among others, have primarily concentrated on the single public good model. The question of private provision of multiple pure public goods has been briefly addressed in the past by Bergstrom, Blume and Varian (1986) and analyzed recently in more detail by Cornes and Itaya (2004). These papers, however, concentrate on the issue of "distribution neutrality" (see also Warr, 1983) in such a setting, i.e. under what conditions total provision is independent of the agents' resource endowment distribution.

Some of the questions that we address in this paper relate to the literature on "united charities", i.e. centralized fund-raising agencies that provide multiple public goods (Bilodeau, 1992 and Bilodeau and Slivinski, 1997). Bilodeau (1992) looks at a situation where individuals are allowed to contribute simultaneously to individual charities (providing a single public good) and to a united charity and shows that "contributing only to the united fund" can be a subgame perfect equilibrium if the fund

[2] Cornes and Itaya (2004) show that the public good mix arising in a Nash equilibrium is suboptimal relative to the first best. The joint contribution scheme raises social welfare by re-aligning provision towards the social optimum.

plays after all contributions are made and is able to offset direct contributions to individual charities. Bilodeau and Slivinski (1997) also look at non-profit organizations that may provide bundles of public goods but focus on the public good supply side, namely the rivalry between such organizations for donations showing that, under some conditions, they will tend to specialize in the public goods they provide since diversification diminishes the equilibrium level of contributions they will collect. These papers are mostly interested in total or individual contribution levels and the possible commitment issues[3] arising due to the fact that charities' own preferences towards the public goods may differ from contributors' ones. As such they do not provide a formal analysis of the individual and social welfare implications of joint versus separate provision schemes as well as the explicit role of heterogeneity that we focus on here.

More generally, our results also relate to the literature on private goods bundling (Adams and Yellen, 1976; McAfee et al., 1989) or excludable public goods bundling such as TV, access to electronic libraries, etc. (Fang and Norman, 2004). Fang and Norman analyze the constrained optimal mechanism of public good provision in a model where agents' valuations are private information and show that it involves bundling. The intuition is that bundling alleviates the free riding problem in large economies by decreasing the extent of use exclusions. In contrast, our results apply to pure (non-excludable) public goods where agents' preferences are public knowledge. Relatedly, Heal (2001) looks at bundling of public with private goods (e.g. by real estate developers) and shows that this can increase welfare by ensuring efficient level of public good provision while Flores (1999) demonstrates how bundling of voting proposals about public goods provision can be used to ensure passage or defeat.

The value of analyzing potential problems pertaining to private provision of public goods can be especially high in development economy contexts where government failure and institutional weaknesses often limit public provision (Besley and Ghatak, 2004). As recently argued by Durlauf and Fafchamps (2004), "In poor countries, there are many situations in which the state could, theoretically, intervene to provide a public good, but where it is unable to do so because its tax base and its capacity to organize are limited. Collective action could serve as a substitute for the state." The importance of studying the effects of inequality in the wealth or land distribution on the public good provision process has been widely acknowledged in some recent applied work (e.g. Bardhan et al., 2002 and Bardhan and Dayton-Johnson, 2002). Our paper contributes to this literature by proposing an additional mechanism (not relying on direct wealth redistribution) which might lead to social welfare enhancement in unequal societies in situations where public goods are privately provided.

2 Model

We consider private provision of multiple public goods as in Bergstrom, Blume and Varian (1986) and Cornes and Itaya (2004). There are two agents indexed by $i = 1, 2$. Each agent divides her wealth $M_i > 0$ between private consumption, c_i and contributions towards two pure public goods G and H. Let $M = M_1 + M_2$. Preferences are given by $u^i(G, H, c_i)$ where u^i is strictly increasing and concave in all three arguments. Following the literature, the agents are assumed to act simultaneously and non-cooperatively and we use Nash equilibrium as the solution concept.

As explained in the introduction, we are interested in comparing public good provision and welfare under two different public good provision schemes - a "separate provision scheme" (hereafter SP) where the two agents are free to contribute any amount they like (including zero) to each of the two public

[3] We concentrate instead on the "redistributional" welfare aspects of using an ex-post optimal fixed sharing rule.

goods and a "joint provision scheme" (hereafter JP) where each agent has to decide on a total non-negative contribution which is then distributed between the two public goods using a pre-announced fixed sharing rule under full commitment[4]. We describe the two provision schemes in more detail below.

2.1 Separate Provision (SP)

Under separate provision, each agent decides on his contributions $g_i \geq 0$ and $h_i \geq 0$ towards the two public goods. The total supplied amount of the first public good is then given by $G = g_1 + g_2$ and the total supplied amount of the second public good is given by $H = h_1 + h_2$. Let $z_i = g_i + h_i$ denote total contribution by agent i. The maximization problem of agent i is:

$$\max_{g_i,h_i,c_i} \; u^i(g_i + G_{-i}, h_i + H_{-i}, c_i) \tag{SP}$$
$$s.t. \; c_i + g_i + h_i = M_i$$
$$g_i, \; h_i, \; c_i \geq 0$$

where, as usual, G_{-i} and H_{-i} denote total contributions by all agents different from i. Existence of a Nash equilibrium in the above setting is easily shown as in Bergstrom et al. (1986).

2.2 Joint Provision (JP)

Under the joint provision scheme the two agents do not contribute separately to each public good as in the previous case, but instead decide on a total contribution amount $z_i \geq 0$. These total contributions are divided by a third party,[5] hereafter referred to as the "public good provider", using the following sharing rule which is announced to the agents before they make their contribution decisions. The share of the total contribution, $Z = z_1 + z_2$, which goes to the public good G is given by $\lambda \in [0, 1]$ and the share of total contribution that goes to H is given by $1 - \lambda$. In the analysis below we assume that the provider chooses λ to maximize the equilibrium social surplus (defined as sum of utilities[6]) obtained when agents take λ as given. We also discuss the case when λ is chosen to maximize total provision Z. Given λ, agent i's optimization problem is:

$$\max_{z_i,c_i} \; u^i(\lambda(z_i + Z_{-i}), (1 - \lambda)(z_i + Z_{-i}), c_i) \tag{JP}$$
$$s.t. \; z_i + c_i = M_i$$
$$c_i, \; z_i \geq 0$$

2.3 Equilibrium Conditions

The above formulation allows for both homogeneous and heterogeneous preferences. We show that this has crucial implications for our results. In order to be able to obtain analytical results we restrict attention to a general class of functional forms for u defined below. Note that the requirement that

[4] See section 4.3 below for some further discussion on this.

[5] Who might be United Way, the Red Cross, the elders in a commune, the village headman, the head of a religious community, etc.

[6] Although we derive our results under a utilitarian social welfare function, it will become clear that the main findings go through under any function featuring a small enough degree of inequality aversion.

preferences belong to this class is only a (potentially strong) *sufficient but not necessary* condition for the results we derive. Specifically let us define:

Definition 1

We will say that the function $u(G, H, c)$ belongs to the set of functions Φ if:

(1) $u(G, H, c) = f(G, H) + v(c)$ where the functions f and v are continuous, strictly increasing and concave, $f_{12} \geq 0$, and f and v satisfy Inada boundary conditions in their respective arguments, namely marginal utility in G, H, c is $-\infty$ as they go to zero and 0 as they go to infinity.

(2) either $u(G, H, c)$ is homothetic[7] *or f satisfies the following property (P1): $f_1 = f_2$ implies $Gf_{11} = Hf_{22}$ where subscripts denote partial derivatives.*

The above definition encompasses a wide variety of functional forms commonly used in economics. It turns out that the homothetic case is easiest to work with but most of our results generalize for the case when f satisfies property (P1). The property requires that, whenever the first partials of f are equalized, the same is implied for the product of each argument and its second partial derivative. Among the functions that satisfy it are Cobb-Douglas, any $f(G, H) = F_1(G) + F_2(H)$ with F_i, $i = 1, 2$ being log, power, exponential or any other function such that $xf''(x)$ is a monotonic function of $f'(x)$, as well as other commonly used functions. In the particular case when u takes the Cobb-Douglas form, $u = \alpha \ln G + \beta \ln H + \ln c$ (also used by Cornes and Itaya, 2004) we are able to provide a full analytical characterization of the contribution equilibria which is exhibited in section 5. Notice also that the above definition allows for preferences in which one of the public goods is always "liked more" by an agent i.e. its marginal utility is higher for the same amount provided. For example, in the separable case $f(G, H) = F_1(G) + F_2(H)$ this can be achieved by assuming $F_1^{'}(x) < F_2^{'}(x)$ for all x. Finally, the Inada and concavity assumptions ensure that both public goods G and H will be always provided in a positive amount in equilibrium.

From now on assume that $u^i \in \Phi$, $i = 1, 2$. The first order conditions in the separate provision case are given by:

$$\begin{aligned} &[g_1] \quad f_1^1(G, H) \leq v^{1'}(M_1 - g_1 - h_1) \\ &[h_1] \quad f_2^1(G, H) \leq v^{1'}(M_1 - g_1 - h_1) \\ &[g_2] \quad f_1^2(G, H) \leq v^{2'}(M_2 - g_2 - h_2) \\ &[h_2] \quad f_2^2(G, H) \leq v^{2'}(M_2 - g_2 - h_2) \end{aligned} \tag{1}$$

where the inequalities are strict when the agent is not contributing, i.e. g_i or h_i is zero. The subscripts denote partial derivatives and the superscripts denote agents.

Similarly, the first order conditions under joint provision are:

$$\begin{aligned} &[z_1] \quad \lambda f_1^1(\lambda Z, (1-\lambda)Z) + (1-\lambda) f_2^1(\lambda Z, (1-\lambda)Z) \leq v^{1'}(M_1 - z_1) \\ &[z_2] \quad \lambda f_1^2(\lambda Z, (1-\lambda)Z) + (1-\lambda) f_2^2(\lambda Z, (1-\lambda)Z) \leq v^{2'}(M_2 - z_2) \end{aligned} \tag{2}$$

again with strict inequalities when an agent is not contributing. We assume that the above system has a unique solution for a given λ. Notice that in the case when u^i is homothetic[8], the individual

[7] The only property that we require is actually that demand for the private good c is a constant fraction of income, i.e. the public good contribution, z does not depend on λ in the joint provision case.

[8] Again, this is a sufficient condition. Any function for which $\frac{\partial z_i}{\partial \lambda} = 0$ will do.

contributions, z_1 and z_2 are independent of λ since the demand for the private good, $M_i - z_i$ is a constant fraction of M_i no matter how the rest of the wealth is split on the other two goods. As we show below, this property enables us to obtain a very easy analytical characterization of the contribution equilibrium. In general, the contributions depend on the sharing rule which is another factor creating a wedge between the provision levels under the joint and separate schemes. We show that the main result of the paper that social welfare under joint provision can be higher than under separate provision remains robust under these alternative specifications.

3 Homogeneous Preferences for Public Goods

We first characterize the equilibria which arise when the agents' preferences towards the two public goods are homogenous, i.e. $u^1(G,H,c) = u^2(G,H,c) = u(G,H,c)$. We then compare the welfare consequences in the separate provision case and the joint provision case. Without loss of generality assume $M_1 \geq M_2$.

Proposition 1
Suppose $u(G,H,c) \in \Phi$. Under preference homogeneity,
(a) there exist only two types of equilibria under both the separate and joint provision schemes:

(1) both agents contribute to both goods if their wealths are not too unequal

(2) only the richer agent contributes if agents wealths are sufficiently unequal

(b) The individual and total contribution amounts to the public goods as well as the public goods provision levels are the same under both separate and joint provision.
(c) The social surplus achieved under the optimal sharing rule λ^* is the same under both separate and joint provision.
(d) Under joint provision, the sharing rule λ^* maximizing social surplus also maximizes total provision, Z and λ^* equals the implied "share", $\lambda^S = \frac{G^S}{G^S+H^S}$ where G^S and H^S are the equilibrium provision levels under separate provision.

Proof:
(a) Start with the SP case. From the first order conditions in (1) and using the fact that G, H are strictly positive in equilibrium, by inspection it is clear that either all four FOCs hold at equality or, given that $M_1 \geq M_2$, only the first two (it is impossible to have one or three equalities). Thus either both agents contribute to both public goods each or only the richer agent contributes to both public goods ($g_2 = h_2 = 0$). The latter case is an equilibrium whenever M_2 is small enough relative to M_1 such that $v'(M_2) > v'(M_1 - g_1^S - h_1^S)$ where g_1^S and h_1^S solve

$$f_1(g_1,h_1) = f_2(g_1,h_1) = v'(M_1 - g_1 - h_1)$$

i.e. when the poorer agent's marginal utility of an extra unit private consumption is higher than that of contributing this unit to either public good given the other agent's contributions. The same logic applies in the JP case where either both or only the first FOC can hold with equality. We derive the exact condition on M_1 and M_2 for the Cobb-Douglas case in section 5.

(b)-(d) We will show that the separate provision equilibrium $(g_1^S, g_2^S, h_1^S, h_2^S)$ is also the equilibrium under joint provision when $\lambda = \lambda^S$. We then show that λ^S is also the value of λ that maximizes social welfare (and total provision) under JP.

First, it is clear from the SP FOCs, (1) that $(\lambda^S, z_1^S, z_2^S)$ where $z_1^S = g_1^S + g_2^S$ and $z_2^S = h_1^S + h_2^S$ satisfy the JP FOCs, (2). Thus, the separate provision contributions represent the joint provision equilibrium under $\lambda = \lambda^S$ as defined in the proposition statement. All we need to show now to finish the proof is that $\lambda^* = \lambda^S$ will indeed be optimally chosen by the provider under joint provision. Let $W^J(\lambda)$ denote equilibrium social surplus under JP. We have:

$$W^J(\lambda) = 2f(\lambda Z^*, (1-\lambda)Z^*) + v(M_1 - z_1^*) + v(M_2 - z_2^*)$$

where Z^*, z_1^* and z_2^* are the equilibrium values as functions of λ. Taking the derivative with respect to λ we have:

$$\begin{aligned}\frac{\partial W^J(\lambda)}{\partial \lambda} &= 2[Z^* f_1 + \lambda f_1 \frac{dZ^*}{d\lambda} - Z^* f_2 + (1-\lambda) f_2 \frac{dZ^*}{d\lambda}] - v'(M_1 - z_1^*)\frac{dz_1^*}{d\lambda} - v'(M_2 - z_2^*)\frac{dz_2^*}{d\lambda} = \\ &= 2[Z^* f_1 - Z^* f_2] + [\lambda f_1 + (1-\lambda) f_2]\frac{dZ^*}{d\lambda}\end{aligned}$$

using (2). Consider first the case when u is homothetic or, in general, satisfies $\frac{dz_1^*}{d\lambda} = \frac{dz_2^*}{d\lambda} = 0$. The above derivative is then zero at $\lambda = \lambda^S$ since $f_1 = f_2$ from (1). Under our assumptions W^J is concave in λ ($\frac{\partial^2 W^J}{\partial \lambda^2} < 0$ since f_{11}, f_{22} and $-f_{12} < 0$) thus λ^S maximizes social surplus under JP. Since in this case it is also true that $\frac{dZ^*}{d\lambda} = 0$, λ^S also maximizes total contribution[9]. This completes the proof for this case.

Now look at the more general case of $u \in \Phi$ but z_1^* and z_2^* depend on λ. We will prove that under (P1) we have $\frac{dZ^*}{d\lambda} = 0$ at $\lambda = \lambda^S$ which will once again give us the desired result. Consider the case when both players contribute under JP (the other case is analogous). Differentiate the FOCs, (2) with respect to λ and solve for $\frac{dZ^*}{d\lambda} = \frac{dz_1^*}{d\lambda} + \frac{dz_2^*}{d\lambda}$ to obtain:

$$\frac{dZ^*}{d\lambda} = \frac{2[f_2 - f_1 + (1-\lambda)Z^* f_{22} - \lambda Z^* f_{11}]}{2[\lambda^2 f_{11} + (1-\lambda)^2 f_{22}] + v''(M_1 - z_1^*) + v''(M_2 - z_2^*)}$$

Using that $f_1 = f_2$ at λ^S (from (1)) and the fact that under (P1) this implies that $(1-\lambda)Z^* f_{22} = \lambda Z^* f_{11}$ at λ^S, we obtain that $\frac{dZ^*}{d\lambda}|_{\lambda=\lambda^S} = 0$ indeed which completes the proof.□

Notice that, with homogeneous preferences of the class we study, our results imply that each agent will be equally well off under both the separate and joint contribution schemes i.e. there does not exist a scenario where bundling leads to welfare improvement over separate provision. Note that this is true for any degree of wealth inequality between the agents. The intuition is that with homogeneous preferences both agents have the same demand functions for the two public goods. The sharing rule, λ^* maximizing social surplus thus reflects the same marginal valuations of the agents toward each of the two goods. Hence, bundling leads to the same total contribution and provision levels as separate provision.

In addition, we have shown that the optimal sharing rule under JP also maximizes the total contribution that can be obtained. Intuitively, distorting agents' choice away from the preferred (and

[9] λ^S cannot be a minimum of $Z^*(\lambda)$ since this will imply that higher provision amount can be achieved in JP compared to SP which is impossible given that under homogeneous preferences all JP allocations are clearly feasible under SP.

feasible) SP allocation would diminish the amount they provide in the case when Z depends on λ. Overall, the proposition implies that preference heterogeneity is a necessary condition for a bundling policy to generate a different equilibrium compared to the case of separate provision.

4 Heterogenous Preferences For Public Goods

Assume now that the agents have different preferences. For expositional clarity concentrate on the case $v^1 = v^2$ and $f^1(G, H) = f^2(H, G)$. We will show that the degree of wealth inequality between the agents is crucial for whether bundling can achieve a social welfare improvement. For a relatively general class of preferences we show that, under sufficiently high wealth inequality, joint welfare is higher under joint provision compared to under separate provision. In contrast, when the two agents have equal wealths, we demonstrate that they will be worse off under joint provision as it distorts the agents' contributions to a suboptimal allocation. These general results are complemented by section 5 where we provide a complete analytical and graphical analysis for the case of Cobb-Douglas preferences where some additional interesting findings can be obtained.

4.1 High Wealth Inequality

Assume that M_2 is sufficiently smaller than M_1 so that agent 2's optimal choice is not to contribute in equilibrium[10] under both the separate and joint provision regimes. The Inada assumptions on v ($v(0) = \infty$) and continuity guarantee the existence of a range of values[11] for M_2 with a left bound of 0 where this property must hold. The Inada conditions on f in turn imply that the richer agent (agent 1) must contribute to both public goods in any such equilibria. For example, in the separate provision case this would hold for M_1 and M_2 such that

$$v'(M_2) > f_1(h_1^S, g_1^S) \text{ and } v'(M_2) > f_2(h_1^S, g_1^S)$$

where g_1^S and h_1^S are the solutions to

$$f_1(g_1, h_1) = f_2(g_1, h_1) = v'(M_1 - g_1 - h_1)$$

Similar conditions can be derived for the JP case. The exact conditions on M_1 and M_2 needed are shown for the Cobb-Douglas case in section 5.

We prove the following result for high wealth inequality.

Proposition 2

Suppose u^1 and $u^2 \in \Phi$. Under preference heterogeneity and sufficiently high wealth inequality:

(a) Total contributions under JP are maximized at $\lambda = \lambda^S$ with λ^S defined as in Proposition 1.

(b) Total contributions are (weakly) lower under JP compared to under SP.

[10]Under our assumptions this guarantees uniqueness of the Nash equilibrium (see Cornes and Itaya, 2004).

[11]The respective intervals may be different in general for the SP and JP schemes in which case we take the smaller one.

(c) The joint surplus maximizing share, λ^* under joint provision is in general different from the implied "share" λ^S thus the contribution allocations differ between the two regimes. More precisely, λ^* is between λ^S and 1/2.

(d) Social welfare under JP at λ^* is higher than social welfare under SP with the richer agent being worse off while the poor being better off.

Proof:

(a)-(b) Consider first the case of u^1, u^2 homothetic. Then we know that agent 1's equilibrium contribution, $z_1 = Z^*$ is independent of λ. Clearly, $Z^S = G^S + H^S$ solves the JP FOCs at $\lambda = \lambda^S$ thus we must have $Z^* = Z^S$ for all λ. Total contributions under JP are constant and equal to those under SP.

Consider now the non-homothetic functions in the set Φ. Let us first show that $\frac{dZ^*}{d\lambda} = 0$ at $\lambda = \lambda^S$ i.e. total contributions are once again maximized at the implied SP share as under preference homogeneity. Differentiate the FOC of agent 1 under JP with respect to λ to obtain:

$$\frac{dZ^*}{d\lambda} = \frac{f_2 - f_1 + (1-\lambda)Z^* f_{22} - \lambda Z^* f_{11}}{\lambda^2 f_{11} + (1-\lambda)^2 f_{22} + v''(M_1 - Z^*)}$$

Notice that at $\lambda = \lambda^S$ we have that $Z^* = Z^S$ is the solution to (2) using (1). In addition, $f_1(\lambda^S Z^*, (1-\lambda^S)Z^*) = f_2(\lambda^S Z^*, (1-\lambda^S)Z^*)$ from the SP FOCs, (1) and, by (P1) we also have $(1-\lambda)Z^* f_{22} = \lambda Z^* f_{11}$ as well. Overall, this implies that

$$\frac{dZ^*}{d\lambda}|_{\lambda=\lambda^S} = 0$$

i.e. total contributions under JP are indeed maximized at $\lambda = \lambda^S$.

(c) Suppose without loss of generality that $\lambda^S \leq 1/2$. Consider social surplus under joint provision when only agent 1 contributes ($Z = z_1$):

$$W^J(\lambda) = f(\lambda Z^*, (1-\lambda)Z^*) + f((1-\lambda)Z^*, \lambda Z^*) + v(M_1 - Z^*) + v(M_2)$$

Differentiate with respect to λ and evaluate at $\lambda = \lambda^S$ to obtain (remember $\frac{dZ^*}{d\lambda}|_{\lambda=\lambda^S} = 0$):

$$\frac{\partial W^J(\lambda)}{\partial \lambda}|_{\lambda=\lambda^S} = Z^*[f_1(\lambda Z^*, (1-\lambda)Z^*) - f_2(\lambda Z^*, (1-\lambda)Z^*) - f_1((1-\lambda)Z^*, \lambda Z^*) + f_2((1-\lambda)Z^*, \lambda Z^*)]$$

Holding Z^* fixed, define $m(\lambda) = f_1(\lambda Z^*, (1-\lambda)Z^*) + f_2((1-\lambda)Z^*, \lambda Z^*)$. We can then write:

$$\frac{\partial W^J(\lambda)}{\partial \lambda}|_{\lambda=\lambda^S} = m(\lambda) - m(1-\lambda)$$

By our assumptions on f we have that $m(\lambda)$ is strictly decreasing in λ thus for $\lambda^S < 1/2$ we have $\frac{\partial W^J(\lambda)}{\partial \lambda}|_{\lambda=\lambda^S} > 0$ i.e. the joint surplus maximizing sharing rule, λ^* must be higher than λ^S. Conversely, if $\lambda^S > 1/2$ we must have $\lambda^* < \lambda^S$. Only if $\lambda^S = 1/2$ we have $\lambda^* = \lambda^S$. These results imply that the contribution allocations under SP and JP will differ in general.

In the homothetic case where $\frac{dZ^*}{d\lambda} = 0$ for all λ we can actually solve analytically for λ^* and show that $\lambda^* = 1/2$. Notice that $\lambda = 1/2$ makes $\frac{\partial W^J(\lambda)}{\partial \lambda}$ zero (it is the same expression as $\frac{\partial W^J(\lambda)}{\partial \lambda}|_{\lambda=\lambda^S}$ but for all λ). Given that $W^J(\lambda)$ is concave under our assumptions ($\frac{\partial^2 W^J}{\partial \lambda^2} < 0$ since f_{11}, f_{22} and $-f_{12} < 0$ and $\frac{dZ^*}{d\lambda} = 0$) this implies that joint surplus is maximized at $\lambda^* = 1/2$ under homotheticity.

(d) The first part is obvious given the definition of λ^* and the fact that $\lambda^* \neq \lambda^S$ (notice that we would obtain the same allocation under JP and SP if we set $\lambda = \lambda^S$). To see that both agents cannot be made better off under JP, remember that only the richer agent contributes in this case. His utility maximization problem under SP can be then written as:

$$\max_{\lambda, Z} \; u^1 = f(\lambda Z, (1-\lambda)Z) + v(M_1 - Z) \tag{3}$$

where we have replaced the original choice variables g_1 and h_1 by $\lambda = \frac{g_1}{g_1+h_1}$ and $Z = g_1 + h_1$. Clearly, for a given λ, the problem of agent 1 under joint provision is a constrained version of the above problem, so he would be always weakly worse off under JP as long as agent 2 does not contribute. Given that W^J is higher, it must be the case that the poorer agent is made better off by bundling.□

The intuition that λ^S maximizes total contributions under joint provision in the case of high wealth inequality is that any other sharing λ distorts the sole contributing agent's optimal allocation thus lowering his average marginal utility $\lambda f_1 + (1-\lambda) f_2$ (see (3)) for a given contributed amount. The optimal response of the agent is then to increase private consumption which would lower v'. Joint provision cannot achieve higher total contribution level compared to separate provision because the more "egalitarian" sharing rule (see part (c) above) distorts agent 1's incentives to contribute. Agent 1 would like to contribute more to the good he likes more e.g. H but is not allowed to do so without contributing to the good he likes less which leads to lower provision of H under joint provision.

The proposition shows that the optimal sharing rule under joint provision will be in general different from that under separate provision. For example, if $\lambda^S < 1/2$ (agent 1 likes good G less, e.g. $F^{1\prime}(x) < F^{2\prime}(x)$ for $f(x,y) = F^1(x) + F^2(y)$) then by the symmetry of f this implies that agent 2 likes good H more. The optimal sharing rule under JP then moves provision away from the richer agent's preferred mix implying different equilibrium provision allocations between SP and JP unlike in the homogeneous preferences case.

The welfare results occur because the more egalitarian sharing rule under JP plays a "redistribution" function tilting the provision levels of the two goods more in line with agent 2's preferences given that most of the resources come from the richer agent. This decreases the welfare of agent 1. The welfare effect for agent 2 is not unambiguous in general because the direct positive effect of a preferable public good mix may not be enough to offset the negative effect of the reduced public good contributions. It turns out, however, that for sufficiently high wealth inequality the positive effect dominates the negative and agent 2 is better off. We show in section 5 that the general flavor of these results carries through even in equilibria where agent 2 contributes.

Propositions 2 implies the following important corollary

Corollary 1

For sufficiently high wealth inequality social welfare is strictly higher under joint provision compared to under separate provision while total contribution is the same under homothetic preferences or lower under non-homothetic $u \in \Phi$.

The result that social surplus under joint provision can be strictly higher than social surplus under separate provision is the main result of the paper. It demonstrates that, under preference heterogeneity and when there are multiple pure public goods, a policy of public good bundling can be welfare improving despite a (weakly) lower total contribution level.

4.2 Low Wealth Inequality

In this section we compare the properties of the contribution equilibria under joint and separate provision when the two agents have equal wealths, $M_1 = M_2 = M$ and contrast them with the high inequality results from above. By continuity all qualitative results carry over for a sufficiently low degree of wealth inequality. We show that high wealth inequality is a necessary condition for bundling to be able to raise joint welfare - under an egalitarian wealth distribution both agents are in fact worse off under joint provision.

For analytical tractability, we need to impose some additional sufficient conditions on u. Suppose that $f(G,H)$ is separable in the two public goods, i.e. $f^1(G,H) = F_1(G) + F_2(H)$ and $f^2(G,H) = F_2(G)+F_1(H)$. Suppose also that $F_1'(x) < F_2'(x)$ for all x, i.e. for a given provision level the two agents always "prefer" different public goods - agent 1 prefers H while agent 2 prefers G where by "prefer" we mean obtaining higher marginal utility per unit contributed. We will only look for symmetric equilibria.

Start with the separate provision case. When $M_1 = M_2$, inspection of (1) shows that only two of the FOCs must hold at equality. i.e. each agent contributes only to the public good he values more:

$$\begin{aligned} F_1'(G) &< F_2'(H) = v'(M-H) \\ F_1'(H) &< F_2'(G) = v'(M-G) \end{aligned}$$

The above implies that in equilibrium $G^* = H^* = Z^S/2$ where Z^S solves

$$F_2'(\frac{Z}{2}) = v'(M - \frac{Z}{2}) \tag{4}$$

Under joint provision, symmetry implies that both FOCs must hold at equality and $z_1 = z_2 = \frac{Z^*}{2}$. We prove the following result.

Proposition 3

Under the above assumptions on u and $M_1 = M_2$ we have:

(a) Total contributions under JP are strictly lower than under SP.

(b) Both agents are worse off under JP compared to under SP.

Proof:

(a) In a symmetric JP equilibrium both agents contribute, thus we must have from the FOCs:

$$\begin{aligned} \lambda F_1'(\lambda Z) + (1-\lambda)F_2'((1-\lambda)Z) &= v'(M - Z/2) \\ \lambda F_2'(\lambda Z) + (1-\lambda)F_1'((1-\lambda)Z) &= v'(M - Z/2) \end{aligned}$$

which is possible only if[12] $\lambda = 1/2$. Thus total contributions, Z^* under JP must solve:

$$\frac{1}{2}[F_1'(\frac{Z}{2}) + F_2'(\frac{Z}{2})] = v'(M - \frac{Z}{2}) \tag{5}$$

Compare this with (4) above. Given our assumption that $F_2'(x) > F_1'(x)$ for all x, it is clear that at Z^S the lefthand side of (5) is smaller than its righthand side thus its solution must be some $Z^* < Z^S$.

[12] In the case of logarithmic F_1 and F_2 the two FOCs are independent of λ but still $\lambda = 1/2$ is joint welfare maximizing (see section 5).

(b) Notice that both agents earn the same utility under the above equilibria. Under separate provision we have:

$$W^S = 2[F_1(\frac{Z^S}{2}) + F_2(\frac{Z^S}{2}) + v(M - \frac{Z^S}{2})]$$

while social surplus under joint provision is

$$W^J = 2[F_1(\frac{Z^*}{2}) + F_2(\frac{Z^*}{2}) + v(M - \frac{Z^*}{2})]$$

The latter must be lower than the former since contributing $Z^*/2$ to their favored good was a feasible choice for each agent in the SP regime given the zero contribution of the other agent but they chose the higher contribution $Z^S/2$ instead. Another way to show this is to note that $F_2'(\frac{Z^*}{2}) > v'(M - \frac{Z^*}{2})$, i.e. contributing more to the preferred good is optimal for each agent under SP.□

Bundling causes harm at low levels of inequality since it forces the agents to contribute to both goods while optimally they would contribute only to one of them. This acts as a tax, lowering the marginal utility of contribution for each agent which leads to less amount contributed and lower individual and social welfare.

4.3 Discussion

The intuition why social surplus under separate provision is higher than social surplus under joint provision at low wealth inequality while the opposite is true at high inequality is that, when both players are relatively equal contributors, the negative distortionary effect of the equal sharing rule leading to lower contributions is relatively strong. In contrast, at high inequality when the poorer agent does not contribute, the more equal sharing of total contributions benefits her by more than the loss of welfare of the richer agent and joint surplus actually increases with joint provision. Basically what happens under high inequality and SP is that the richer agent "has his way" in terms of the mix of public goods provided (the poorer agent cannot offset this since he has very little wealth to start with) and this leaves agent 2 much worse off compared to under joint provision which features more preferable (from the viewpoint of the poorer agent) provision levels ratio. By "catering for the poorer agent" joint provision is thus able to improve social welfare under high wealth inequality by undoing to some extent the provision bias towards the richer agent's preferences coming from the inequality in resources.

The results that bundled contribution can never lead to a Pareto improvement over separate contributions and that actually at low levels of inequality joint provision can be welfare reducing to both agents relative to separate provision indicate that a joint provision policy can be defended on social optimality grounds only for high levels of wealth inequality and in a transferable utility framework.

An important implication of our results is the difference between maximizing total provision and maximizing social welfare in the second best environments[13] considered here. Notice that under high inequality (see Corollary 1) it is possible to have *higher* total contribution but *lower* social welfare under separate compared to under joint provision. This has policy significance - wealth redistributions aimed to increase total contributions may actually lead to lower social welfare and vice versa. While

[13]Notice that the JP scheme cannot attain the first best as total contributions are lower than under the SP Nash equilibrium which we know has a sub-optimal contribution level.

it is clear that both the SP and JP regimes feature suboptimal contribution and social surplus levels, one needs to be mindful of the possible inconsistency of policies aimed to raising contribution levels versus those aimed at raising joint welfare as neither of these necessarily implies the other.

Our results were derived in a two-player framework for simplicity but it is clear that they would hold in more general situations with higher number of potential contributors. For example, the high inequality scenario in which we have shown that bundling raises social welfare can be easily reproduced with more than two agents as long as their wealths are unequal enough so that only the richest agent contributes. Similarly, our main result remains valid by considering economies consisting of a big number of replicas of our original two agents and looking at equilibria where the same agents are treated symmetrically. Of course, there exist more general cases with $n > 2$ players that our results do not automatically extend to. Nevertheless, the main intuition that bundling could improve social welfare in situations with preference heterogeneity and high resource inequality by changing the provision mix towards the first best would still remain valid as long as the preference heterogeneity is related to the wealth distribution in a way such that richer and poorer agents prefer different public goods. Still, a more complete formal analysis of the multiple agent case could be a fruitful extension and reveal additional interesting results.

Remember that in the non-homothetic case total contributions collected under bundling depend on the announced share, λ. We saw above that setting $\lambda = \lambda^S$ would lead to higher total contributed amount than under the optimal share λ^*. In our analysis we have assumed that the public good provider is able to fully commit to the pre-announced sharing rule. If this were not the case, it is possible to have a time-inconsistency problem - the provider may announce λ^S to obtain more contributions but then actually share according to λ^* which will lead to higher social welfare. A potential extension of our results in this direction, perhaps using arguments as in Kydland and Prescott (1977) can be of interest.

5 The Cobb-Douglas Case

In this section we assume that preferences take the Cobb-Douglas form (which belongs to our set Φ):

$$u^i(G, H, c_i) = \alpha_i \ln G + \beta_i \ln H + \ln c_i$$

It turns out that a complete analytical characterization of the separate and joint provision equilibria is possible in this case. We use it to further illustrate the intuition and sharpen our general results from the previous sections. In the homogeneous preferences case we set $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$ while heterogeneous preferences are represented by $\alpha_1 = \beta_1 = \alpha$ and $\beta_1 = \alpha_2 = \beta$. Without loss of generality we assume $\beta \geq \alpha$ thus good H is "liked more" by both agents under homogeneous preferences, while with heterogeneous preferences agent 1 likes H more but agent 2 likes G more. Notice also that the Cobb-Douglas form is homothetic, thus the equilibrium z_1 and z_2 under joint provision are independent of λ which simplifies the analytics tremendously.

In the homogeneous preferences case Proposition 1 applies so the SP equilibrium is equivalent to the JP equilibrium under the optimal λ which in this case is easily shown to be $\lambda^* = \frac{\alpha}{\alpha+\beta}$. The optimal sharing rule λ maximizing social surplus is independent of the equilibrium provision and agents' endowments. From the FOCs one can derive that both agents contribute under both SP and JP as long as $M_2 > \frac{M_1}{1+\alpha+\beta}$.

Concentrate now on the heterogeneous preferences case where we can obtain a sharper characterization of the resulting equilibria than in the general analysis above. We suppose $\beta > \alpha$ and maintain our assumption that $M_1 \geq M_2$.

5.1 Separate Provision

First, for $\alpha \neq \beta$ it is easy to see from (1) that there is no equilibrium where all $g_i, h_i > 0$. Thus, at least one of the agent will not contribute to at least one of the goods in equilibrium. Enumerating all possible cases for the FOCs holding as equality or inequality, it turns out that only the following three types of equilibria are possible depending on the parameters of the model.

Case S.I. Equilibrium where agent 1 contributes to both goods and agent 2 does not contribute to any.

Case S.II. Equilibrium where agent 2 does not contribute to H and agent 1 contributes to both goods.

Case S.III. Equilibrium where agent 1 does not contribute to G and agent 2 does not contribute to H.

Solving the linear systems of FOCs for the three cases is an easy but tedious task so we omit the details[14]. The following tables fully characterize the solutions under separate provision in the three possible types of equilibria as well as the parameter restrictions under which the latter occur:

Table 1: Individual and Total Contributions Under Separate Provision

Case	Parameter Restriction	z_1	z_2	Z^S
S.I	$0 \leq M_2 \leq \frac{\alpha M}{(1+\beta)(\alpha+\beta)}$	$\frac{(\alpha+\beta)M_1}{1+\alpha+\beta}$	0	$\frac{(\alpha+\beta)M_1}{1+\alpha+\beta}$
S.II	$\frac{\alpha M}{(1+\beta)(\alpha+\beta)} < M_2 < \frac{\alpha M}{\alpha+\beta}$	$\frac{(\alpha+\alpha\beta+\beta^2)M_1-\beta M_2}{(1+\beta)(\alpha+\beta)}$	$\frac{\beta(1+\alpha+\beta)M_2-\alpha M_1}{(1+\beta)(\alpha+\beta)}$	$\frac{\beta M}{1+\beta}$
S.III	$\frac{\alpha M}{\alpha+\beta} \leq M_2 \leq \frac{M}{2}$	$\frac{\beta}{1+\beta}M_1$	$\frac{\beta}{1+\beta}M_2$	$\frac{\beta M}{1+\beta}$

Table 2: Public Good Provision and Social Surplus Under Separate Provision

Case	G	H	Social Surplus, W^S
S.I	$\frac{\alpha M_1}{1+\alpha+\beta}$	$\frac{\beta M_1}{1+\alpha+\beta}$	$(\alpha+\beta)\ln\frac{\alpha\beta M_1^2}{(1+\alpha+\beta)^2} + \ln\frac{M_1 M_2}{1+\alpha+\beta}$
S.II	$\frac{\alpha\beta M}{(1+\beta)(\alpha+\beta)}$	$\frac{\beta^2 M}{(1+\beta)(\alpha+\beta)}$	$(\alpha+\beta)\ln\frac{\alpha\beta^3 M^2}{(1+\beta)^2(\alpha+\beta)^2} + \ln\frac{\alpha\beta M^2}{(1+\beta)^2(\alpha+\beta)^2}$
S.III	$\frac{\beta}{1+\beta}M_2$	$\frac{\beta}{1+\beta}M_1$	$(\alpha+\beta)\ln\frac{\beta^2 M_1 M_2}{(1+\beta)^2} + \ln\frac{M_1 M_2}{(1+\beta)^2}$

Notice that, for given preference heterogeneity, the nature of the equilibrium which obtains depends on the relative inequality in agents' resource levels M_1 and M_2. For highly diverse wealths (case S.I) only the richer agent contributes to both goods. For intermediate wealth heterogeneity (case S.II) agent 1 still contributes to both goods but now agent 2 also contributes albeit only to the good he likes more (good G). Finally, when wealths are relatively similar (case S.III) each agent contributes only to the good they like more.

[14] However, they are readily available from the authors upon request.

5.2 Joint Provision

Consider now the joint provision case. It is easy to show that the optimal sharing rule that will be chosen by the public goods provider in this case is $\lambda^* = 1/2$ since the contributions are independent of λ in the Cobb-Douglas form and thus maximizing joint surplus is equivalent to maximizing $(\alpha + \beta)[\ln \lambda + \ln(1-\lambda)]$. The optimal sharing rule under heterogeneous preferences is thus to split total contribution in half[15] between the two public goods. The intuition comes from the fact that agents are treated equally by the provider who maximizes the sum of utilities and the fact that agent 1's preferences are a "mirror image" of agent 2's preferences with respect to the two public goods. We fix $\lambda = \lambda^*$ and proceed with characterizing the possible equilibria under joint provision. Looking at the first order conditions (2), there are two possible types of equilibria:

Case J.I. Equilibrium where agent 1 contributes to both goods and agent 2 does not contribute to any ($z_1 > 0,\ z_2 = 0$)

Case J.II. Equilibrium where both agents contribute ($z_1,\ z_2 > 0$)

It is easy to establish that the first equilibrium occurs whenever $M_2 \leq \frac{M}{2+\alpha+\beta}$, while the second occurs if the opposite is true. The following tables characterize the two equilibria under joint provision.

Table 3: Individual and Total Contributions Under Joint Provision

Case	Parameter Restriction	z_1	z_2	Z^J
J.I	$0 \leq M_2 \leq \frac{M}{2+\alpha+\beta}$	$\frac{\alpha+\beta}{1+\alpha+\beta} M_1$	0	$\frac{(\alpha+\beta)M_1}{1+\alpha+\beta}$
J.II	$\frac{M}{2+\alpha+\beta} < M_2 \leq \frac{M}{2}$	$\frac{(1+\alpha+\beta)M_1 - M_2}{2+\alpha+\beta}$	$\frac{(1+\alpha+\beta)M_2 - M_1}{2+\alpha+\beta}$	$\frac{(\alpha+\beta)M}{2+\alpha+\beta}$

Table 4: Public Good Provision and Social Surplus Under Joint Provision

Case	$G = \lambda^* Z^J$	$H = (1-\lambda^*)Z^J$	Social Surplus, W^J
J.I	$\frac{(\alpha+\beta)M_1}{2(1+\alpha+\beta)}$	$\frac{(\alpha+\beta)M_1}{2(1+\alpha+\beta)}$	$2(\alpha+\beta)\ln \frac{(\alpha+\beta)M_1}{2(1+\alpha+\beta)} + \ln \frac{M_1 M_2}{1+\alpha+\beta}$
J.II	$\frac{(\alpha+\beta)M}{2(2+\alpha+\beta)}$	$\frac{(\alpha+\beta)M}{2(2+\alpha+\beta)}$	$2(\alpha+\beta)\ln \frac{(\alpha+\beta)M}{2(2+\alpha+\beta)} + 2\ln \frac{M}{2+\alpha+\beta}$

5.3 Public Good Provision and Welfare Under Separate vs. Joint Provision

Since the Cobb-Douglas function is a particular example of homothetic preferences belonging to the set Φ, we know that all results established in Propositions 1-3 still apply. However, in addition to this, we are now able to provide a complete characterization of the properties of the contribution equilibria for *any degree* of wealth inequality to supplement the limiting results from the previous section. The analytical results are illustrated on figures 1-2 obtained computationally.

Let us start with some general properties of the contribution equilibria with Cobb-Douglas preferences that hold for any M_1 and M_2. We can look at the effect of increasing wealth inequality by raising M_1 while holding total wealth M fixed.

Proposition CD1

Under preference heterogeneity,
(a) total contributions under SP, Z^S are (weakly) higher than those under JP, Z^J for any M_1, M_2.
(b) There exists a threshold level of wealth inequality such that social surplus under SP

[15] Simple algebra shows that the same equal sharing rule is optimal under the first best.

is strictly higher than social surplus under JP for inequality below the threshold and the opposite is true for inequality above the threshold.
(c) For any level of inequality, the welfare of the richer agent is lower under JP compared to under SP. For high enough wealth inequality the welfare of the poorer agent (agent 2) is higher under JP compared to under SP.

Proof: (see Appendix)

The intuition why joint provision cannot achieve higher total contribution level compared to separate provision is that the equal sharing rule distorts the agents' incentives to contribute. The richer agent is worse off under joint provision for any degree of wealth inequality. The intuition behind the welfare results in (b) and (c) was already explained in section 4. Notice that the welfare result in (b) does not require that agent 2 be a non-contributor (as assumed in section 4) - fig. 1b shows that it is possible that both agents contribute under both regimes and still $W^J > W^S$. The analytical tractability of the C-D case allows us to be able to characterize the equilibrium for intermediate inequality levels in addition to the two extremes (high and low inequality) considered in section 4.

The next proposition summarizes some additional results that apply for this case of widely used preferences.

Proposition CD2

(a) The provision level of the public good that the richer agent values more (good H) is (weakly) lower under JP compared to under SP.
(b) The contribution level of the poorer agent under JP is weakly lower than under SP.
(c) Under both JP and SP total contributions (Z^J and Z^S) are maximized under perfect inequality ($M_1 = M$) and $Z^S - Z^J$ is (weakly) decreasing in wealth inequality.
(d) W^J is maximized at intermediate degree of inequality while W^S could be maximized at either perfect equality or intermediate inequality.

Proof: (see Appendix)

Agent 1 would like to contribute relatively more to the good he likes more (good H) but is not allowed to do so which leads to lower provision of H under joint provision. Agent 2 contributes less in total under JP since he is "forced" to contribute to good H which he would never contribute to under separate provision. Agent 1's total contribution under JP may be higher or lower compared to under SP depending on whether the negative effect (on provision) of distorted incentives or the positive effect of the diminished contribution of agent 2 dominate[16].

The intuition for the fact that total provision under both contribution schemes is maximized at perfect inequality is that, when only one agent can contribute, there is no free riding problem and the total amount provided reaches the first best level. Note also that Z^J and Z^S are independent of the resource distribution for certain parameter ranges when the set of contributors does not change by the redistribution as in the "distribution neutrality" results discussed in detail in Bergstrom et al. (1986) and Cornes and Itaya (2004). Total contribution is the same under both regimes for high

[16]The reduction in agent 2's contribution causes the provision levels of the public goods to be "too low for agent 1's taste" so he needs to compensate for this by contributing more. The same argument does not go the other way because, as usual in such settings, the agent with lower marginal valuations (the poorer agent) has stronger free riding incentives - the "exploitation of the rich by the poor" effect as named colorfully by Olson (1965).

wealth inequality since only the richer agent contributes and since, under homothetic preferences, his private consumption (and hence his total contribution) is a constant share of his endowment.

Part (d) shows that social welfare is in general a non-monotonic function of wealth inequality under both separate and joint contribution thus extending the results of Bardhan et al. (2002) for a multiple public good framework. Bardhan et al. look at impure public goods and analyze the effects of inequality in private inputs that are complementary in production with collective inputs. They find that inequality between contributors and non-contributors is typically socially optimal. The intuition for the optimality of inequality comes from the trade-off between two effects working in opposite directions. On the one hand higher inequality leads to a relative decrease in free-riding as one agent becomes the predominant provider while on the other hand it leads to lower social welfare due to the concavity of agents' preferences.

See figs. 1a-1b for graphical illustration of the results in Propositions CD1 and CD2. The figures describe how public good provision, individual and total contributions and welfare vary as function of M_1 holding total wealth M fixed, which is equivalent to varying the degree of wealth inequality between perfect equality ($M_1 = M/2$) and perfect inequality ($M_1 = M$). Notice that public good provision (good G) can be non-monotonic in wealth inequality in the SP regime, decreasing at low inequality while all of G is provided by agent 2, constant when both agents provide it (case S.II), and increasing in M_1 for high wealth inequality when agent 1 is the sole provider. The level of H is increasing in M_1 under SP as agent 1 is the only contributor to this good.

6 Conclusions

There are numerous real life situations where agents contribute voluntarily to more than one public goods. In this paper we examine such a scenario and study the effects of different contribution mechanisms on public good provision and welfare. Once we introduce the case of multiple public goods, a crucial issue is the optimal mix of the public goods provided. In the first part of the paper we show that in the absence of preference heterogeneity amongst the agents, a separate contribution scheme, where the agents contribute individually to the public goods, is equivalent in terms of contributions and welfare to a joint contributions scheme where individual contributions are shared using a fixed rule among the public goods.

The comparison between the two schemes becomes interesting after one moves towards more realism and incorporates heterogeneity amongst the agents. We consider the role of preference and wealth heterogeneity and characterize the different consequences which arise as a result (see also Clark and Kanbur, 2004). We find that in the presence of such heterogeneities the mix of the different public goods is not optimal. The degree of heterogeneity drives the contribution amounts of the agents. We propose a simple mechanism, namely, the bundling of contributions towards the public goods, which might be socially welfare enhancing. We find that this is indeed the case that when wealth heterogeneity is high.

To sum up, as Cornes and Itaya (2004) also point out, voluntary contributions to multiple public goods suffer from inefficiencies of scale as well as mix. Our analysis explores whether a simple solution, i.e. bundling of various public good could help the situation. The results suggest that while the inefficiencies pertaining to the scale may worsen under bundling, their mix can be improved, leading to greater aggregate welfare. It is worth mentioning that we do not assume any informational advantage for the providers (who dictate the share in which the public good will be divided in the joint case). It might well be the case that they might have more relevant information that the donors do not possess.

In that scenario it is even more likely that the bundling scheme will be preferred since the onus of information acquisition then falls on the donors in the separate provision scheme.

Apart from contributing to the existing theoretical work on voluntary public goods contribution, our results have policy implications for situations where agents contribute simultaneously to multiple public goods. We have shown that wealth redistributions aimed to increase total contributions may actually lead to lower social welfare and vice versa. Bundling public good contributions was demonstrated to be socially optimal only for high levels of inequality, thus different contributions schemes may have to be chosen in different situations depending on the degree of preference and resource heterogeneity among agents. Our analysis thus has potential implications for the design of institutions that function on voluntarily donated funds. Charities, NGOs, even research institutions rely on public donations which are then channelled into various uses which have public goods characteristics. This paper provides a framework that can be used to understand when it might be optimal to use bundling and when to have use-specific contributions.

The fact that bundling cannot generate a Pareto improvement over separate provision has potentially interesting political economy implications. Remember that under high inequality the rich agent is actually worse off under joint provision. Depending on whether the median voter in a democratic political system or a dictator otherwise could be classified as "rich" or "poor" in the context of our stylized model, such social welfare improving policies may or may not be implementable in practice.

It could be also useful to study the welfare effects of the joint provision scheme under different public goods technologies, as in many real world situations. For example, one might think of technologies such as best-shot or weaker- or-weakest link (see Cornes, 1993). Similarly there has been some work on introducing dynamics in voluntary public good provision games (Gradstein, 1992 and Varian, 1994). We have also adopted a complete information framework where the agents' preferences are common knowledge. One might investigate the role of dynamics in a multiple public goods setting and the implications of incomplete information. Are there policies in such a setting, like we have shown in a simultaneous game setting, which might lead to welfare improvements? We hope to address some of these issues in future work.

References

[1] Adams, W. and J. Yellen, (1976), "Commodity Bundling and the Burden of Monopoly", *Quarterly Journal of Economics*, 90, pp. 475-498.

[2] Andreoni, J. (1988), "Privately Provided Public Goods in a Large Economy: The Limits of Altruism." *Journal of Public Economics*, v.35, pp. 57-73.

[3] Bardhan, P. and J. Dayton Johnson, (2002), "Inequality and Conservation on the Local Commons: A Theoretical Exercise" *Economic Journal*, Vol. 112, pp. 577-602.

[4] Bardhan, P., M. Ghatak and A. Karaivanov, (2002), "Inequality, Market Imperfections, and the Voluntary Provision of Collective Goods", in P. Bardhan, S. Bowles and J. M. Baland (ed.) in *Economic Inequality, Collective Action, and Environmental Sustainability*, Princeton University Press, Princeton.

[5] Besley, T. and M. Ghatak, (2003), "Public Goods and Economic Development", in A. Banerjee, R. Benabou, and D. Mookherjee (ed.), *Policies for Poverty Alleviation*, Oxford University Press.

[6] Bergstrom, T., L. Blume and H. Varian, (1986), "On the Private Provision of Public Goods", *Journal of Public Economics*, Vol. 29, pp.25-49.

[7] Bilodeau, M., (1992), "Voluntary Contributions to United Charities", *Journal of Public Economics*, 48, pp. 119-133.

[8] Bilodeau, M. and A. Slivinski, (1997), "Rival Charities", *Journal of Public Economics*, 66, pp. 449-467.

[9] Clark, S. and R. Kanbur, (2004), "Stable Partnerships, Matching and Local Public Goods", *European Economic Review,* 48, pp. 905-925.

[10] Cornes, R. (1993), "Dyke Maintenance and Other Stories: Some Neglected Types of Public Goods", *Quarterly Journal of Economics*, 108, pp. 259-271.

[11] Cornes, R. and J. Itaya, (2004), "Models with Two or More Public Goods", Department of Economics Research Paper Number 896, The University of Melbourne.

[12] Cornes, R. and T. Sandler, (1985), "The simple analytics of pure public good provision, *Economica*, 52, pp.103-116.

[13] Dreze, J. and A. Sen, (1999), *India: Economic Development and Social Opportunity*. Oxford University Press.

[14] Durlauf, S. and M. Fafchamps, (2004), "Social Capital", Forthcoming in *Handbook of Economic Growth*, ed by S. Durlauf and P. Aghion.

[15] Fang, H. and P. Norman, (2004), "An Efficiency Rationale for Bundling of Public Goods", Department of Economics Working Paper, Yale University.

[16] Flores, N. (1999), "The Importance of Agenda and Willingness To Pay", working paper 99-30, University of Colorado.

[17] Gradstein, M. (1992), "Time Dynamics and Incomplete Information in the Private Provision of Public Goods", *Journal of Political Economy*, 100, pp. 581-597.

[18] Heal, G. (2001), "Bundling Public and Private Goods", manuscript, Columbia University.

[19] Kydland, Fynn E., and Edward C. Prescott (1977): "Rules Rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, 85(3):473-491

[20] McAfee, P., J. McMillan and M. Whinston, (1989), "Multiproduct Monopoly, Commodity Bundling and Correlation of Values", *Quarterly Journal of Economics*, 104, pp. 371-384.

[21] Ostrom, E. (1990), *Governing the Commons: The Evolution of Institutions for Collective Action.* New York: Cambridge University Press.

[22] Varian, H. (1994), "Sequential Contributions to Public Goods", *Journal of Public Economics*, 53, pp. 165-186.

[23] Warr, P. (1983), "The Private Provision of Public Goods is Independent of the Distribution of Income", *Economics Letters*, 13, pp. 207-211.

7 Appendix

Proof of Proposition CD1

(a) Follows immediately since $\frac{\beta}{1+\beta} \geq \frac{\alpha+\beta}{2+\alpha+\beta}$ (see tables 1-4).

(b) We show first that at perfect equality i.e. $M_1 = M_2 = M/2$, we have $W^S \geq W^J$ with equality only if $\alpha = \beta$. To see this notice that (using tables 1 and 3), $W^{S.III} \geq W^{J.II}$ is equivalent to:

$$(\alpha+\beta)\ln\frac{\beta}{1+\beta} + \ln\frac{1}{1+\beta} \geq (\alpha+\beta)\ln\frac{\alpha+\beta}{2+\alpha+\beta} + \ln\frac{2}{2+\alpha+\beta}$$

Call $\alpha+\beta = A$. Notice that the r.h.s. is a function of A only while the left hand side is decreasing in α holding A fixed. Thus, for given A, the l.h.s. is minimized at $\alpha = \beta = A/2$ and then it actually equals the r.h.s. Thus for any A and any $\alpha < \beta$ the l.h.s. is strictly larger than the r.h.s. i.e. $W^S > W^J$ at perfect wealth equality.

Now consider the case of high wealth inequality, i.e. $M_2 \leq \frac{\alpha M_1}{\beta(1+\alpha+\beta)}$ or $M_1 \geq \frac{\beta(1+\alpha+\beta)M}{(1+\beta)(\alpha+\beta)}$ (cases S.I and J.I). Notice that $sign(W^J - W^S)$ is then equal to $sign(\frac{(\alpha+\beta)^2}{4} - \alpha\beta)$ and the latter is always positive as long as $\alpha < \beta$. Observe that this also implies that $W^J > W^S$ for $M_2 \leq \frac{\alpha M_1}{\beta}$, i.e. for $M_1 \geq \frac{\beta M}{\alpha+\beta}$ (fact 1), since $W^{S.II}$ equals $W^{S.I}$ evaluated at $M_2 = \frac{\alpha M_1}{\beta(1+\alpha+\beta)}$ holding M fixed. Now look at W^S and W^J as a function of M_1 holding M fixed. Notice that W^S is strictly decreasing in M_1 for case S.I., i.e. $M_1 \in [\frac{M}{2}, \frac{\beta M}{\alpha+\beta}]$ starting at a level higher than W^J (fact 2) and then constant for $M_1 \in [\frac{\beta M}{\alpha+\beta}, \frac{\beta(1+\alpha+\beta)M_1}{(1+\beta)(\alpha+\beta)}]$ (case S.II.). In contrast W^J is constant for all $M_1 \in [\frac{M}{2}, \frac{(1+\alpha+\beta)M}{2+\alpha+\beta}]$ (case J.II) while under case J.I W^J is initially *increasing*[17] in M_1 for $M_1 \in [\frac{(1+\alpha+\beta)M}{2+\alpha+\beta}, \frac{1+2(\alpha+\beta)}{2(1+\alpha+\beta)}M)$[18] and then decreasing in M_1 (fact 3). Given facts 1-3 (see also fig. 2) one can easily convince oneself that W^J and W^S plotted as function of M_1 holding M constant must cross at a unique point $M_1 = m \in (\frac{M}{2}, \frac{\beta M}{\alpha+\beta})$ which is the threshold from the proposition statement.

(c) Under S.I., J.I must hold so $u_1^{J.I} \leq u_1^{S.I}$ is equivalent to

$$\frac{\alpha\ln\alpha + \beta\ln\beta}{2} \geq \frac{\alpha+\beta}{2}\ln(\frac{\alpha+\beta}{2}) \tag{6}$$

which is true since the function $x\ln x$ is convex for $x > 0$. Under S.II. we need to show that $u_1^{S.II}$ is bigger or equal to u_2^J achieved in both cases J.I and J.II since both are possible. Since under S.II we have that $M \geq \frac{(1+\beta)(\alpha+\beta)M_1}{\beta(1+\alpha+\beta)}$, we have $u_1^{S.II} \geq \alpha\ln\frac{\alpha M_1}{1+\alpha+\beta} + \beta\ln\frac{\beta M_1}{1+\alpha+\beta} + \ln\frac{M_1}{1+\alpha+\beta} = u_1^{S.I}$ and we already know this is not smaller than u_1^J under J.I. Now suppose S.II and J.II hold instead. Given that $\frac{\beta}{(1+\beta)(\alpha+\beta)} \geq \frac{1}{2+\alpha+\beta}$ we have $u_1^{S.II} \geq \alpha\ln\frac{\alpha M}{2+\alpha+\beta} + \beta\ln\frac{\beta M}{2+\alpha+\beta} + \ln\frac{M}{2+\alpha+\beta} \geq u_1^{J.II}$ because of (6). Now, suppose S.III. and J.I hold. Given that $\frac{1}{1+\beta} \geq \frac{1}{1+\alpha+\beta}$, we have $u_1^{S.III} \geq \alpha\ln\frac{\alpha M_1}{1+\alpha+\beta} + \beta\ln\frac{\beta M_1}{1+\alpha+\beta} + \ln\frac{M_1}{1+\alpha+\beta} \geq u_1^{J.I}$ because of (6). The same logic works under S.III and J.II because $\frac{1}{1+\beta} > \frac{1}{2+\alpha+\beta}$. The fact that the poor agent must be better off under high inequality follows from part (b). □

Proof of Proposition CD2

(a) For $M_2 \in [0, \frac{\alpha M_1}{\beta(1+\alpha+\beta)}]$, the statement holds since $\frac{(\alpha+\beta)}{2(1+\alpha+\beta)} \leq \frac{\beta}{1+\alpha+\beta}$. Similarly, for $M_2 \in [\frac{\alpha M_1}{\beta}, M_1]$ the statement is true since $\frac{\beta M_1}{1+\beta} \geq \frac{\beta M}{2(1+\beta)} > \frac{(\alpha+\beta)M}{2(2+\alpha+\beta)}$ and also $\frac{\beta}{1+\beta} > \frac{\alpha+\beta}{2(1+\alpha+\beta)}$. Finally,

[17] Notice that W^S can be also increasing for some range of M_1 above $\frac{\beta(1+\alpha+\beta)M}{(1+\beta)(\alpha+\beta)}$.

[18] This is easily seen differentiating W^J with respect to M_1 after substituting $M_2 = M - M_1$.

when $M_2 \in (\frac{\alpha M_1}{\beta(1+\alpha+\beta)}, \frac{\alpha M_1}{\beta}]$, $H^{JP} < H^{SP}$ since then $H^{SP} = \frac{\beta^2 M}{(1+\beta)(\alpha+\beta)} > \frac{\beta M_1}{1+\alpha+\beta} \geq \frac{(\alpha+\beta)M_1}{2(1+\alpha+\beta)}$ and also since it is easily verified that $\frac{\beta^2}{(1+\beta)(\alpha+\beta)} \geq \frac{(\alpha+\beta)}{2(2+\alpha+\beta)}$ for $\beta \geq \alpha$.

(b) Suppose first we are in case S.II. Simple algebra then shows that $z_2^{SP} = \frac{\beta(1+\alpha+\beta)M_2-\alpha M_1}{(1+\beta)(\alpha+\beta)} \geq \max\{0, \frac{(1+\alpha+\beta)M_2-M_1}{2+\alpha+\beta}\} = z_2^{JP}$ for $\beta \geq \alpha$. Similarly, in case S.III. we have that $z_2^{SP} = \frac{\beta M_2}{1+\beta} \geq \frac{(1+\alpha+\beta)M_2-M_1}{2+\alpha+\beta}$ is equivalent to $(1+\beta)M_1 \geq (1+\alpha)M_2$ which is always true given our assumptions.

(c) Under JP, Z^J is independent of the wealth inequality under J.II and strictly increases in inequality (as M_1 increases) under J.I. Since Z^J is a continuous function, as we vary M_1 holding M fixed, it is clear that it must be maximized at $M_1 = M$. Similarly, under separate provision Z^S is independent of the wealth distribution under S.II and S.III (lower inequality) and strictly increases in inequality under S.I. By its continuity it is also maximized at $M_1 = M$, i.e. perfect inequality. Since $\frac{\beta}{1+\beta} \geq \frac{\alpha+\beta}{2+\alpha+\beta}$ it is clear from tables 1 and 3 that for $M_2 \in (\frac{\alpha M}{(1+\beta)(\alpha+\beta)}, \frac{M}{2}]$ (lower inequality) we have $Z^S > Z^J$ while for $M_2 \in [0, \frac{\alpha M}{(1+\beta)(\alpha+\beta)}]$ (higher inequality) we have $Z^S = Z^J = \frac{(\alpha+\beta)M_1}{1+\alpha+\beta}$. Holding M constant, increasing inequality is equivalent to decreasing M_2 relative to M_1 i.e. moving up the rows in tables 1 and 3. Both Z^S and Z^J are continuous functions of $M_1$and M_2 and thus their difference is also continuous function of inequality. To show that $Z^S - Z^J$ is decreasing in wealth inequality, notice that for low inequality, i.e. $M_2 \in [\max\{\frac{\alpha M}{\alpha+\beta}, \frac{M}{2+\alpha+\beta}\}, \frac{M}{2}]$, we have that $Z^S - Z^J$ is constant and strictly positive for $\beta > \alpha$, for intermediate inequality, $M_2 \in [\frac{\alpha M}{(1+\beta)(\alpha+\beta)}, \frac{M}{2+\alpha+\beta}]$, Z^S is constant while Z^J increases in M_1 towards $\frac{\beta M}{1+\beta}$, and for high inequality, $M_2 \in [0, \frac{\alpha M}{(1+\beta)(\alpha+\beta)})$ the total contributions are equal.

(d) The first part follows directly from our findings above - W^J is maximized at $M_1 = \frac{1+2(\alpha+\beta)}{2(1+\alpha+\beta)}M \equiv m^*$. W^S could also have a local maximum at m^* depending on the parameter values for α and β. The global maximum of W^S then is achieved at either $M_1 = M/2$ or $M_1 = m^*$.□

Fig. 1a - C-D, Varying Wealth Inequality (M=1, α=0.3, β=0.7)

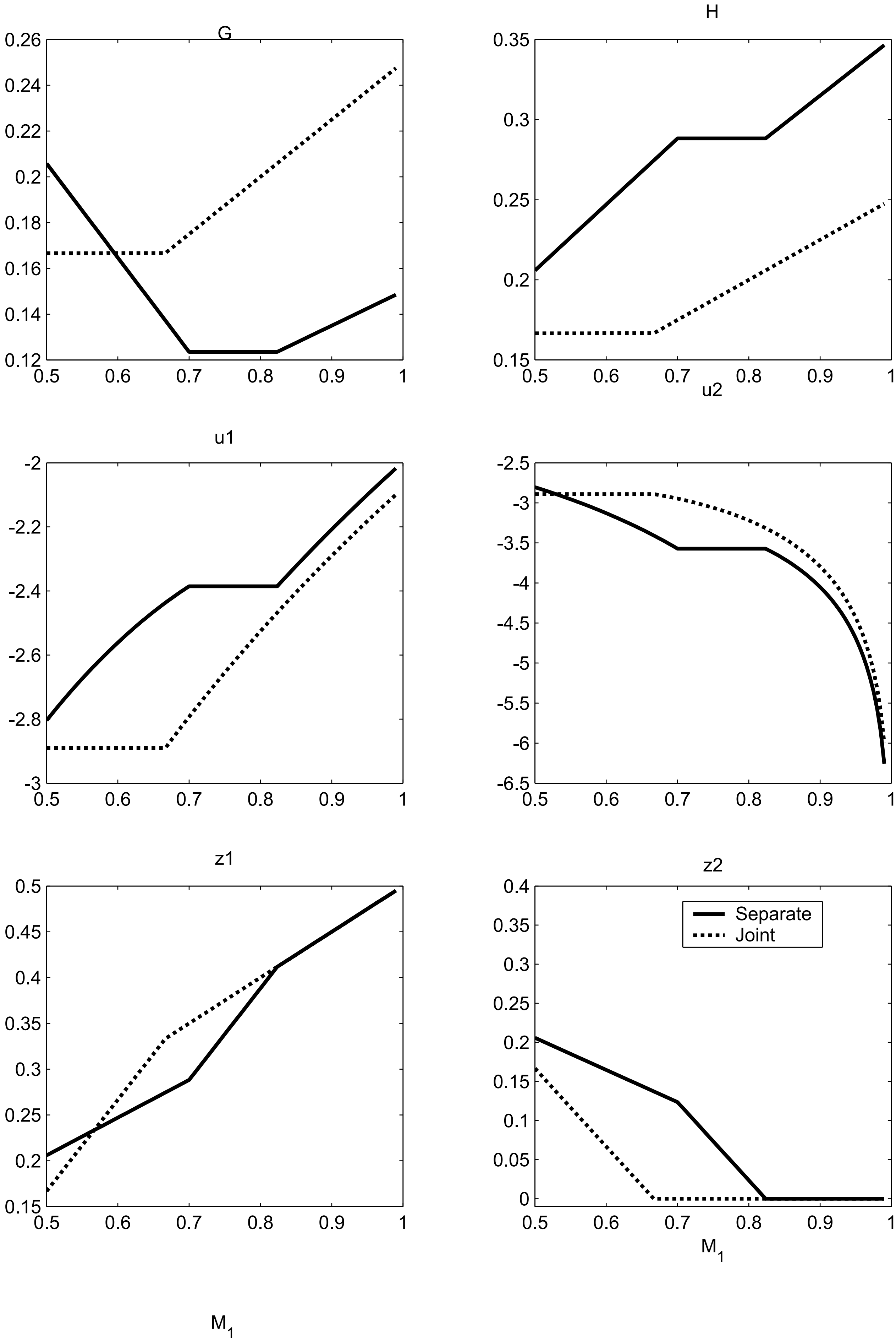

Fig. 1b - C-D, Varying Wealth Inequality (M=1, α=0.3, β=0.7)

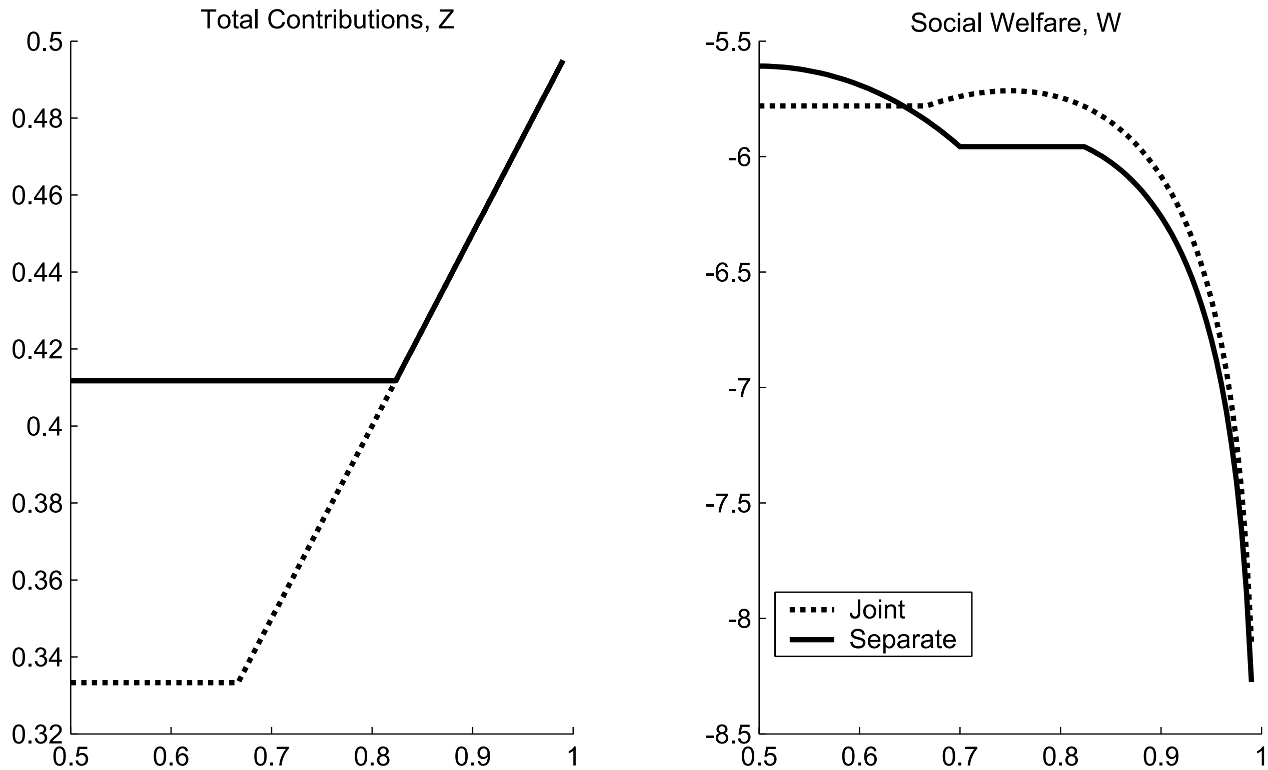

Fig. 2 - C-D, Varying Preference Heterogeneity ($\alpha+\beta$=1, M_1=0.65, M_2=0.35)