



Job mobility and careers in firms

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Abstract

This paper presents a theoretical model that combines employers learning about worker productivity, human capital acquisition, job-assignment and resolution of worker uncertainty regarding disutility of work from a job, to show how widely documented findings on both wage and promotion dynamics and turnover can be captured in a single set-up. Specifically we show how our model can capture results such as; probability of turnover decreases with labor market experience, wage changes during job changes is more in earlier periods, serial correlation in wages and probability of promotion increases in wages, amongst others.

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1. Introduction

In this paper we build a model by combining theoretical constructs to address the empirical findings in two areas: wage and promotion dynamics within firms and the turnover literature. A theoretical model combining these two areas results in significant insights, apart from deriving the results from both these areas in a single model. The idea that entry and exit opportunities of workers has influence on how employers within the firm structure their wage and promotion policies are derived in this paper.

The model we develop for the above purpose is a stylized model and some simplifying assumptions are made. But this allows us to make the model analytically tractable and at the same time address some important findings in the literature. It is a multi-period model with a hierarchical firm structure with job levels where there is symmetric learning among firms about

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the ability of the workers. Workers derive disutility by working in a firm, and workers accumulate both specific and general human capital?¹ Specific human capital ties workers to their employers while the disutility of work being an “experience good” generates turnover. Specifically, workers gradually learn about their disutility of working in a particular firm as their career proceeds. Thus, they are promoted up the job ladder due to the general and specific human capital they accumulate over time, but their realization of the disutility of work factor may bring about turnover.

With this basic structure our model is able to address a number of results from the empirical literature. For example, we can explain why the probability of separation declines with tenure in the following way. The employer offers a premium to its workers each period over the market wage offer since workers accumulate specific human capital, which is appropriated by the employer. After the disutility of work in a particular firm is realized each period, workers compare their net utility in the current firm with the option of moving to another firm. The workers having a higher realization of the disutility of work in the random draw, on average change firms early in their careers. With tenure, the workers who stay with the firm, on average have a smaller disutility of work in the firm in which they are working. Hence, given that they receive a wage premium, there is a small probability that workers change employers late in their career.

The other results we get are that of a rising and concave wage schedule and that the wage difference during job change declines with the age of the worker. With regard to job levels, we find that there are no definite ports of entry and exit in the job ladder, the higher level jobs are filled predominantly from candidates inside the firm as against from outside, and also other results on wage and promotion dynamics (for example, serial correlation in wages, wage increases predict promotion).

The reasons we think that our model is a positive contribution to the previous theoretical work on careers in firms are the following. First, all the previous models treat the turnover issues separately and thus the wage and promotion dynamics involved are not treated explicitly as part of the analysis. In this paper we deal with the separation issues in an integrated model of the wage and promotion dynamics. Second, another notable aspect of this paper is that this is a more complete analysis compared to the existing theoretical literature since the equilibrium is derived by assuming lifetime utility maximization on the part of workers and profit maximization on the part of firms. Third, the existing models of turnover put a strong emphasis on the financial components of the compensation a worker gets. While financial factors are crucial, an employee can also obtain (positive or negative) utility from other aspects related to the job.² We incorporate this idea into our analysis of mobility patterns. Our model thus provides an important extension of earlier research done on job mobility which has focused solely on financial incentives.

The outline of the paper is as follows. This section is followed by a brief survey of the previous empirical and theoretical literature. Given the vast amount of work that has been done regarding careers in firms we restrict our survey to work closely related to our analysis. For more comprehensive surveys see [Gibbons and Waldman \(1999a\)](#) and [Prendergast \(1999\)](#). Section 3 builds a 3 period model without job levels to easily understand the forces at work which drive the results. In Section 4 we extend the model to multiple job levels with symmetric learning among

¹ The disutility realization is different from the matching models ([Jovanovic, 1979](#)) in the sense that the disutility is privately realized by the worker only while in the matching models, the employer and the firm simultaneously realize their match. Thus there is no asymmetric information regarding this aspect in the standard matching models.

² [Groot and Verberne \(1997\)](#) show empirically that by splitting the mobility pattern into moves to, from, and between bad or good jobs that non-financial aspects are also important in the job mobility decisions of employees. They conclude that employees make an evaluation of a possible trade-off between financial and non-financial compensation when deciding whether or not to change jobs.

employers and derive the findings mentioned above.³ We follow it up with a discussion in Section 5 that extends the model of the previous section and the empirical predictions from our model. In Section 6 we conclude.

2. A brief survey of the previous empirical and theoretical literature

2.1. Empirical findings

The established facts about worker turnover are due to [Parsons \(1977\)](#) and [Mincer and Jovanovic \(1981\)](#). Their main results are that the probability of separation declines with labor-market experience and firm-specific seniority. [Topel and Ward \(1992\)](#) present empirical tests of the [Burdett \(1978\)](#) and [Jovanovic \(1979\)](#) explanations for why the probability of separation is negatively related to labor-market experience and (ultimately) to firm-specific seniority. Topel and Ward find that after controlling for the wage there is a positive relationship between the probability of separation and experience. In another related paper, [Farber \(1997\)](#) has confirmed the following findings: long term employment relationships are common, new jobs end early and the probability of job ending declines with time. [Farber \(1994\)](#) also finds (apart from the above three results) that mobility is strongly positively related to the frequency of job change prior to the start of the job. Lastly, the findings on wage differences in the turnover process are that the difference in wages during job change declines with the age of the worker ([Bartel and Borjas, 1982](#); [Mincer, 1986](#)).

Along with the empirical findings on separation issues, recently there has been some empirical work relating to wage and promotion dynamics inside firms. Two important papers in this genre are that of [Baker et al. \(BGH henceforth\) \(1994a,b\)](#). Regarding job turnover with respect to job levels, they found that there are no definite ports of entry and exit. The other results that we capture from the BGH study are the following: First, serial correlation in wage increases ([BGH, 1994a](#)). Second, higher level jobs are filled by candidates from the internal pool of workers as against from the outside market ([BGH, 1994b](#)). Third, workers who receive larger wage increases early in their stay at one level of the job ladder are promoted more quickly to the next level ([BGH, 1994b](#)). The result that promotions are associated with large wage increases have been empirically verified by [Garhart and Milkovich \(1989\)](#), [Lazear \(1992\)](#) and [McCue \(1996\)](#).

Lastly there is a significant empirical literature in the last decade on theories of wage growth. Early influential papers include [Abraham and Farber \(1987\)](#) and [Altonji and Shakotko \(1987\)](#), both of which conclude that that firm-specific seniority has a minor effect on wage growth. Subsequently there has been work trying to ascertain the exact reason behind wage growth in firms.⁴ Our model shed's light on that empirical debate.

2.2. Theoretical literature

The models of turnover can be divided into two main categories. In the first category are the models in which turnover occurs as a result of the arrival of information about the current job match. These are models in which a job is an “experience good” ([Jovanovic, 1979](#)), where the only way to determine the quality of a particular match is to form the match and “experience” it. These papers provide an explanation for why the probability of separation declines with firm-specific seniority.

³ See [Ghosh \(2002\)](#) for a n- period generalization of this model. Once we analyze the 3 period model with job levels, it should be clear that this model easily generalizes to the n- period.

⁴ See [Topel \(1991\)](#) and [Altonji and Williams \(1998\)](#).

The second category of models are the “pure search-good” models of job change (Burdett, 1978; Mortensen, 1978) where matches dissolve because of the arrival of new information about an alternative prospective match. Burdett (1978) provides an explanation for why the probability of separation declines with labor-market experience with a model where jobs are inspection goods.

Another perspective on the separation issue is the heterogeneous-worker model developed by Blumen et al. (1955). In their model each worker is characterized by a fixed probability of separation, which varies across workers. Other models in this line of literature include MacDonald (1998), who develops a stochastic job matching model and uses it to derive a set of testable restrictions on the conditional probability with which a worker will be observed to change jobs over time. Lazear (1986) analyzes wage-setting and turnover in a model where the worker’s skill has a firm-specific component. In Lazear’s model the probability that the outside employer is informed of the worker’s skill is exogenously given. Worker-firm separations can also be modeled by the asymmetric-learning approach as in Greenwald (1986). He applies Akerlof’s (1970) “lemons” model to the issue of labor mobility.

The turnover literature summarized above does not address the empirical findings on wage and promotion policies within firms. A number of papers in the recent literature investigate what Gibbons and Waldman (1999a) refer to as integrative models, which is a model that combines one or a few approaches to generate results matching the empirical findings.⁵ Gibbons and Waldman (1999b) integrate the two standard ways of modeling the promotion process as job-assignment mechanism: learning and human capital acquisition. They show that a framework that integrates these familiar ideas captures a number of recent empirical findings concerning wage and promotion dynamics inside firms. None of the above papers on integrative models consider the turnover issues. What we attempt to do in this paper is to investigate empirical results from the worker’s separation literature in such an integrative model. Thus our model is the first paper which considers the interaction between these two literatures (job turnover and wage/promotion dynamics) and how it might give rise to additional insights in order to capture empirical results.⁶

2.2.1. An example

Given the different elements that are present in our model, we choose to first provide an example with two periods that will bring out the key element that drives the results. Basically the specific human capital and the disutility of work factor will determine the extent of turnover in our model. In our set-up, firms are perfectly competitive. The production function of the worker is given by $y=X$, except if it is the second period and a worker has stayed with the firm he worked for in the first period whereby it is $y=X+s$. Here ‘ X ’ is the output from the worker corresponding to his general human capital and ‘ s ’ is the specific human capital realization gained from experience of working in a given firm. There is a disutility of working at a firm which is uniform on $[-.5, .5]$. Suppose that the worker has worked for firm e in period 1. We are concerned about the career prospects of the worker in period 2. Let w_e be the wage offered by firm e in period 2 and w_m be the market wage offer. The net utility level of each worker in period 2 is given by $U=(w-d)$, where d is the disutility factor as mentioned above. The worker that is indifferent to staying or leaving his current firm has to have a net utility from the current firm (w_e-d^*) which is equal to the outside wage offer (since the expected outside disutility is zero). Hence, $d^*=w_e-w_m$, as a result of which firm e gets $(d^*+.5)$ workers and the outside firm gets $(.5-d^*)$ workers.

⁵ See Demougin and Siow (1994) and Bernhardt (1995) for two such integrative models.

⁶ Munasinghe (2000) is another paper in this direction but his model explains certain specific facts rather than a more general array of results.

Given that the outside firms are perfectly competitive, the outside wage offer w_m^* will be X .⁷ Profits by firm e will be given by $(X+s-w_e)(d^*+.5)$. Substituting for d^* from above we get $(X+s-w_e)(w_e-X+.5)$. To maximize profits, firm e chooses w_e such that the profit term is maximized, i.e. $Max_{w_e}(X+s-w_e)(w_e-X+.5)$. So the profit maximizing wage w_e^* determines the cut-off level of the workers who stay with the current employer. From the first order condition we get that $w_e^*=X+s/2-0.5/2$. Thus $d^*=(s-0.5)/2$. From above it is clear that as 's' increases, w_e^* increases. Intuitively it means that the original employer who had employed the worker in period 1, values the worker more if the specific human capital is higher and hence tries to compensate the worker by paying him more in order to insure against the realization of the disutility factor. Note, that in this 2-period set-up the outside firms can never realize the benefits of the specific human capital since the worker needs to work in a given firm for at least one period. Hence the market always offers the wage corresponding to the general productivity of the worker, which is X . Similarly for a high 's' the cut-off disutility realization above which the worker leaves the firm is also higher as can be seen from the expression for d^* above. In other words, the higher the 's', the lower the job turnover. Thus, this example illustrates the main ingredient of our model, that is, the tension between the specific human capital and the disutility realization of the workers.

3. The model without job levels

In this Section we develop a 3 period model which sheds insight on the basic mechanisms at work and derive the following results from this set-up: probability of turnover decreases with labor market experience, difference in wages during wage change declines with the age of the worker and workers have an increasing and concave wage schedule.

3.1. Model

There is only one job level. Let us assume without loss of generality that the worker has worked for firm e in period 1. We are concerned about the worker's career over period 2 and period 3. Workers and firms are risk neutral and have a discount rate of zero. In this model we try to capture the wage rates and turnover levels of workers during periods 2 and 3. Let us denote the outside firms as the market. After each period the worker receives simultaneous wage offers from firm e and the market. We assume a Bertrand wage offer game and thus we can consider only the best outside offer from the market to compete with firm e .⁸ Let W_{it} denote the wage earned by a worker i in period t . The net utility level of each worker is given by

$$U = (W_{i2}-D_{i2}) + (W_{i3}-D_{i3}) \quad (1)$$

where D_{it} is the disutility of worker i in a particular firm for the t -th period. Note that this is firm-specific disutility. The disutility is realized after working for a period in a firm. We assume that D_i in

⁷ The solution concept which we use is that of a market-Nash equilibria where given the first period employer's strategy, the market has a strategy which is consistent with what would result from competition among a large number of firms. Similarly, given the market's strategy, the first period employer will maximize her expected profits. The consequence of the market strategy is that the strategy of the market must everywhere be consistent with a zero-expected-profit constraint (See Waldman, 1984).

⁸ In a symmetric learning set-up the assumption of a Bertrand wage offer from the employers is not restrictive. But it is noteworthy that in set-ups of asymmetric information regarding each worker's productivity the nature of the bidding can have significant consequences for the basic results. In that case the employer can observe the outside bid and strategically decide to conceal his information which he reveals through the wage offer.

any period is a draw from a uniform distribution with support $[D_L, D_H]$. Here D_L is negative and D_H is positive, to capture the fact that workers might also derive a positive utility for working in a firm.

The production function of the worker is given by

$$y_{it} = X_{it} + s_i, \quad (2)$$

where X_{it} represents the general human capital acquired by the worker i and s_i is the specific human capital.⁹ s_i takes the value 0 if the worker is working for the first period in a firm and takes the value s if he has worked for at least a period in the firm.

The way the disutility factor comes into play is as follows. The worker works in firm e for period 1. While working for the first period he privately experiences the disutility of working in firm e . When he gets the wage offers from firm e and the market in the beginning of period 2, he takes the disutility into account to calculate his net expected utility in period 2. While working in period 2 in firm e , there is a fraction q of workers who receive a new draw of disutility and for the fraction $(1-q)$ the disutility remains the same as in period 1. This is introduced to capture the idea that frequently there is some exogenous factor that can cause disutility of work to change from one period to the next. As in period 1, they experience the disutility in period 2 which they take into account to calculate the net expected utility in the beginning of period 3 when they receive the wage offers. The workers for whom the disutility remains the same calculate their net expected utility with the same disutility as in period 2, while the rest take their new realization of D_2 and use this value to calculate their net expected utility for period 3.

3.2. Analysis

Every period before bidding for a worker, the firms observe the output of the worker in the past periods. The firm which has already employed the worker for one period has an incentive to offer a mark-up over the market wage offer because of the specific human capital factor. But there is the random realization of the disutility of work factor which propels turnover. Workers for whom this realization is large enough (see the Proof of Proposition 1) may prefer to change firms even with the mark-up in wages offered to them because their net expected utility is higher by doing so. Workers consider their expected net utility for periods 2 and 3 when taking the decision. Their decision is thus in accordance with utility maximization throughout their entire career.

Similarly firms know that workers have different disutilities of work. As long as the specific human capital factor is not too large, there will always be workers on the margin between staying and quitting.¹⁰ We know that the disutility of work is distributed uniformly with mean zero and range $[D_L, D_H]$. The higher the wage offered to workers of a given ability level the greater the number who prefer to stay rather than quit. The firm weighs this factor against the gain in specific and general human capital which the worker acquires over time, which is realized by the firm if they can keep the worker. By considering their expected profits over the entire time period they arrive at the wage offer each period that maximizes their profits.

Our model has a different way of modeling the job-turnover process as compared to the models discussed earlier. For example in Jovanovics' model the output of a worker is composed of the

⁹ The general human capital is typically a function of the innate ability and the tenure in the firm, which we explicitly model in Section 4. For now we assume that $X_{i3} > X_{i2}$.

¹⁰ If the specific human capital is large enough then the employer will ensure that the workers always stay with the employer by providing a high enough premium to the outside wage offer. We are thus assuming that 's' is not so large that this is the case.

worker-firm match which is drawn from a nondegenerate distribution of the worker's productivity across different jobs. The realization of the match is observed simultaneously by both parties and the workers are always paid their marginal products. But in our model the channel by which turnover takes place is through the disutility of work realized by the worker along his career path. The disutility of work realization is privately known by the worker. Thus there is an asymmetry of the learning process. As a result of this firms have to pay a premium to workers (since they realize the gains from specific human capital from workers) in order to compensate workers for their disutility of work realization. Thus incorporating specific human capital (which is not present in Jovanovic's model) creates a tension in the turnover process which in turn helps us to address results on wage dynamics inside the firm.¹¹

Let us consider the wage schedule which is offered to the workers by the firm e . If the firm is employing the worker for consecutive periods then the incentive for the firm to provide a premium to the worker is because of the current period specific human capital. If one of the outside firms is successful in hiring the worker then after one period they can also realize the gains from the specific human capital. This fact is reflected in the outside wage offer. And also the outside wage offer in period 2 captures the fact that the outside firms' offers include the profits which they can earn if the worker does not turnover in period 3. The firm e in order to compete with the outside wage offer also pays that amount to its workers. Note, that since $q < 1$, the probability that the worker has a new realization in the original firm in period 3 is q as against 1 in the outside firms. This asymmetry between the firms is important and the worker also takes this into account while choosing firms.

Below, π_t is the expected profit in period t . $\bar{W}_{i2}(\cdot)$ and $\bar{W}_{i3}(\cdot)$ are the wage offers from the market in period 2 and 3 respectively. Correspondingly W_{i2}^* and W_{i3}^* are the wage offers of firm e in periods 2 and 3. And lastly $F(\cdot)$ is the distribution function of the disutility factor (note that we assume that F is uniform) and P_2 and P_3 are the probability of turnover for period 2 and period 3 respectively. All proofs are in the Appendix.

Proposition 1. The wages received by worker 'i' who stays in a given firm is $W_{i2}^* = \bar{W}_{i2} + \frac{s+D_L(1+F(\frac{s+D_L}{2})(1-q))}{2}$ and $W_{i3}^* = \bar{W}_{i3} + (s + D_L)/2$ for periods 2 and 3 respectively. The probability of turnover for periods 2 and 3 are given by $P_2 = \left\{ 1 - F\left(\left(\frac{W_{i2}^* - \bar{W}_{i2}}{1+F(\frac{s+D_L}{2})(1-q)}\right)\right) \right\}$ and $P_3 = \left\{ 1 - F\left(W_{i3}^* - \bar{W}_{i3}\right) \right\}$ respectively. In equilibrium $\bar{W}_{i2} = X_{i2} + \pi_3$ and $\bar{W}_{i3} = X_{i3}$, where $\pi_3 = \left(\frac{s-D_L}{2}\right)^2 \left[\frac{q}{D_H - D_L} + \frac{(1-q)}{D^* - D_L}\right]$.

3.3. Results

From the above proposition we can derive some results which match the empirical findings in the literature.

Corollary 1.1. $P_2 > P_3$. Probability of turnover decreases with labor market experience. (Parsons, 1977; Mincer and Jovanovic, 1981).

The firms provide a premium to the worker in wages since they can realize the gains from specific human capital from the worker. They cannot reap this if the workers move out from their firm. Given

¹¹ We believe that the matching models are insufficient in capturing the wage and promotion dynamics results and thus our avenue is a simplified way to capture that aspect. But certainly the pure matching models mentioned before are extremely useful in capturing other results related to turnover.

this premium the workers decide to stay or leave the firm depending on the disutility of work. In the second period they know that for a current disutility level there is a positive probability that they have to endure this in the third period also. Thus when they take into account their net utility for their entire career then they are more likely to change jobs. While the third period being the last period of their career, it is more likely that for the wage premium provided by the employer they stay with the current firm compared to the second period. Hence it is more worthwhile in a net expected utility sense to change firms earlier in the career as compared to later.

One thing to note is that if $q=1$ then we would not get the above result.¹² Rather the probability of turnover would remain the same each period. The reason for this is the following. At the beginning of period 2, the wage that the original employer will offer will take into account the current profit and also the future expected profit. The outside firms will also do the same when giving its offer. Now, if the outside employer is successful in hiring the worker in period 2 then in period 3 they also reap the added benefit of the specific human capital. Hence after period 2 the worker yields the same expected profit to both the employer and the outside firms. But this is not so if $q < 1$. In this case the probability that the worker has a new realization in the original firm in period 3 is q as against 1 in the outside firms. This asymmetry between the firms is important and the worker also takes this into account while choosing firms.

Corollary 1.2. *Difference in wages during job change in period 2 is more than that in period 3 (Bartel and Borjas, 1982).*

In period 2, the worker is more likely to turnover, compared to period 3. The gain to the employer for being able to keep the worker in its own firm is due to a single period's extra productivity due to specific human capital. Since this is a joint maximization exercise on both the firm's and the worker's part, the firm knowing this tendency of the worker tries to compensate the worker with a higher wage compared to the market offer in period 2. This gives rise to the fact that the difference in wages during job change in period 2 is more than that in period 3.

Corollary 1.3. *Rising and concave wage schedule for the workers wages (Abraham and Farber, 1987; Topel, 1991).*

We get that if the gains from the specific human capital are not too large then each period the wages typically rise for workers within a firm.¹³ At the beginning of the second period the market wage offer reflects the fact that there is an expected profit from the worker in the third period. And thus the market wage offer is the productivity of the worker in period 2 plus the expected profits from the third period. To compete with the market offer firm e also has to include the expected profits from the worker in the third period in addition to the wage premium $((s + D_L)/2)$ due to the specific human capital factor which the firm e gives to its workers. Thus, competition with the market drives the result that the worker is compensated for the specific human capital factor. While the 3rd period is the last period in the career of the worker and so the outside firms cannot realize the gains from the worker in the future. So the outside wage offer is just the productivity in the 3rd period. Firm e offers only the premium due to the specific human capital factor which they realize in the 3rd period. Thus when a worker gets to period 3 he experiences a rise in his wage schedule due to the increase in the

¹² To check this we can substitute $q=1$ in the wage equations for period 2 and 3 respectively in proposition 1 and find that the difference in the wages offered between the original employer and the outside firms remain the same. Hence both the turnover probabilities are also the same.

¹³ $s < \bar{s}$ where $\bar{s} = 2(X_{i3} - X_{i2}) + D_L$ is a sufficient condition for this to hold. See Proof of Corollary 1.3.

effective ability. On the other hand there is a downward effect on wages due to the one period expected profit factor which workers received in period 2 as against the outside wage offer in period 3 which does not reflect that. The condition given in footnote 11 is a sufficient condition which guarantees that there is a rise in the wage schedule. Thus the results of our model are consistent with the findings due to Altonji and Shakotko (1987) on rising wage schedules which they argue is primarily due to general human capital accumulation.

The concave wage schedule is due to the concavity of the general human capital accumulation function. As we can verify from the proposition that the wage offered is function of the expected output of the worker each period (X_{it}), the general human capital function makes the wage schedule over the career of the worker concave.

4. Model with job levels

We extend the model of the previous section by incorporating job levels and symmetric learning on the part of the employers about workers' ability. Our objective is to show, that we get the results on wage and promotion dynamics also in our model which includes turnover. This section is thus similar in flavor to the analysis in Gibbons and Waldman (1999b), except that in this there is the possibility of turnover, which was not there in their analysis.

4.1. The Model

Worker i 's innate ability is denoted by θ_i and can be either high or low: $\theta_i \in \{\theta_H, \theta_L\}$. A worker's effective ability is a function of the worker's innate ability and the worker's labor market experience. Let t denote calendar time and x_{it} the worker's labor-market experience prior to period t . We assume that worker i 's effective ability in period t is given by

$$\eta_{it} = \theta_i f(x_{it}), f' > 0, \text{ and } f'' < 0. \quad (3)$$

The firm consists of three different jobs, denoted by 1, 2 and 3. If worker i is assigned to job j in period t , then the worker produces

$$y_{ijt} = d_j + c_j(\eta_{it} + \varepsilon_{ijt}) + s_j \quad (4)$$

where d_j and c_j are constants known to all labor market participants and ε_{ijt} is a noise term drawn from a normal distribution with mean 0 and variance σ^2 . s_j takes the value 0 if the worker is working for the 1st period in a firm and takes the value s if he has worked for a period in the firm.

And lastly there is symmetric learning of the workers' abilities by the firms. At the beginning of a worker's career a worker is known to be of innate ability θ_H with probability p_0 and of innate ability θ_L with probability $(1-p_0)$. Learning takes place at the end of each period when the realization of the worker's output for that period becomes common knowledge. Let θ_{it}^e denote the expected innate ability of worker i in period t : $\theta_{it}^e = E(\theta_{it} | z_{it-x}, \dots, z_{it-1})$. From θ_{it}^e we can compute the expected effective ability of worker i in period t :

$$\eta_{it}^e = \theta_{it}^e f(x_{it}). \quad (5)$$

The way the updating process works is as follows. Define $z_{it} = (y_{ijt} - d_j - s_j) / c_j = \eta_{it} + \xi_{ijt}$. That is, z_{it} is the signal about the worker's effective ability that the market extracts from observing the worker's output in period t . Note that the signal z_{it} is independent of job-assignment and thus there is no difference in the rate of learning across jobs.

Let z^x denote the normalized output history $(z_{it-x}, \dots, z_{it-1})$. Let $p = \text{Prob}(\theta = \theta_H / z^x)$ be the probability that the worker has high innate ability given z^x . By Bayes' rule we can update this value by the observation z_{it} in the following way:

$$\text{Prob}(\theta = \theta_H / Z^x, z_{it}) = \frac{ph[z_{it} - \theta_H f(x)]}{ph[z_{it} - \theta_H f(x)] + (1-p)h[z_{it} - \theta_L f(x)]}$$

The learning about a worker's productivity is important in the model because the wages that a worker receives is dependent on the information of the worker's productivity which the firm learns. Let η' denote the effective ability level at which a worker is equally productive at jobs 1 and 2, and η'' denote the effective ability level at which a worker is equally productive at jobs 2 and 3. That is, η' solves $d_1 + c_1\eta = d_2 + c_2\eta$ and η'' solves $d_2 + c_2\eta = d_3 + c_3\eta$. We assume $c_3 > c_2 > c_1 > 0$ and $d_1 > d_2 > d_3 > 0$, and that these parameters are such that $\eta'' > \eta'$. The disutility of work realization and the symmetric learning takes place in the same manner as stated before in Section 3.

During period 1 the worker works for firm e. Learning takes place at the end of each period. As before firms observe z_{it} and thus learn about the innate ability of the worker. While working in period 1 the workers experience their disutility of work in firm e. At the beginning of period 2 firms offer wages simultaneously and allocate workers to job levels. Workers decide which wage offer to accept by taking into account the disutility which they have experienced in the previous period to calculate the net utility. After working in period 2, a worker has a new realization of the disutility of work in firm e with probability q while with probability $(1-q)$ the disutility factor remains the same as before. At the beginning of period 3 the same process takes place as in the beginning of period 2. That is, firms observe the output in period 3 and update their priors about the innate ability of the worker and then decide on the job allocation, and workers observe their disutility from work. Firms simultaneously offer wages and allocate workers to jobs.

Compared to the model in the previous section, now there are job levels and the firms have to decide on the allocation of workers to job levels. The promotion decision of the firm is solely dependent on the expected effective ability since firms cannot behave strategically in this regard because of symmetric learning. The learning aspect of this model drives the wage and promotion dynamics results and the random realization of the disutility of work factor drives the mobility result.

4.2. Analysis

The role of the specific human capital factor is same as in the previous model. The firm which has already employed the worker for a period derives a one period benefit due to the specific human capital which the outside firms are unable to receive. This is independent of the job level to which the worker is assigned. After one period the benefit to the employer and the benefit to the outside firms are the same. Thus the market wage offer reflects the expected future profits from the specific human capital factor. As a worker's career progresses it is less likely for the worker to change jobs given a positive disutility of work realization. This is because when the worker is in the early stages of his career, the disutility which the worker might have to sustain is larger, depending on the years of service left. Hence the firm will take into account this factor while offering the premium. This in turn implies that during the initial stages of the worker's career, the wage premium that is given with respect to the market offer is higher in order to compensate the workers for their higher propensity for turnover. During the later stages of the career, this effect diminishes.

To reduce the number of cases that we need to consider (which does not affect the basic results) we impose the following reasonable parametric restrictions. (i) $\theta_L f(2) > \eta'$ and (ii) $\theta_H f(1) > \eta''$.

The first restriction says that in the third period, none of the remaining workers stay at job 1 and the second restriction implies that none of the worker's is promoted to job 3 in period 2. As before for notational economy we denote $d_j + c_j \eta_{it}$ as X_{ijt} , where X_{ijt} denotes the expected productivity of the i th worker in job j at period t . As before, P_t denotes the probability of turnover in period t and \overline{W}_{ijt} is the market wage offer for worker i in period t corresponding to the job level j . Finally, W_{ijt}^* is the equilibrium wage offer by the employer in period t , corresponding to the job level j .

Proposition 2. In the model with job levels, job assignments, wages and turnover probabilities in period 2 and period 3 are given by

(i) If $\eta_{it}^e < \eta'$ ($\eta' < \eta_{it}^e < \eta''$) then the worker i is assigned to job 1 (job 2) in period 2 and $W_{i12}^* = d_1 + c_1 \eta_{i2}^e + \pi_3 + \left(\frac{s+D_L(1+F(\frac{s+D_L}{2})(1-q))}{2} \right) \left(W_{i22}^* = d_2 + c_2 \eta_{i2}^e + \pi_3 + \left(\frac{s+D_L(1+F(\frac{s+D_L}{2})(1-q))}{2} \right) \right)$.

The probability of turnover is given by $P_2 = \left\{ 1 - F \left(\left(\frac{W_{i2}^* - \overline{W}_{i2}}{1+F(\frac{s+D_L}{2})(1-q)} \right) \right) \right\}$.

(ii) If $\eta' < \eta_{it}^e < \eta''$ ($\eta'' < \eta_{it}^e$) then the worker i is assigned to job 2 (job 3) in period 3 and $W_{i23}^* = d_2 + c_2 \eta_{it}^e + \frac{s+D_L}{2}$ ($W_{i33}^* = d_3 + c_3 \eta_{it}^e + \frac{s+D_L}{2}$). The probability of turnover is given by $P_3 = \left\{ 1 - F(W_{i2}^* - \overline{W}_{i3}) \right\}$.

The first term in the wage function is the expected productivity of the worker in the current period. The second term is the expected profit to the employer if the worker stays in the firm for the remaining period of his career. Note that this takes into account the fact that the worker might turnover from the firm in any future period. The last term is the wage premium offered due to the specific human capital, the benefit of which can be reaped by the employer in the form of increased output since the worker has already worked for one period with him.

The signal z_{it} is independent of job assignment. So there is no difference in the rate of learning across jobs. Thus, there is no way to use job assignment to change the speed of learning about ability, so job assignment is determined by current productive efficiency, which in turn is solely a function of the worker's current effective ability.

The job assignment matters for a worker only in terms of the wage rate he receives. But since this is a symmetric learning model, the market also has the same information on the workers' ability and so it responds by allocating the same job assignment as in firm e . The difference between the market wage offer and the employers' offer, which is the wage premium, is determined only by the specific human capital and hence independent of the job allocation. Basically workers decisions to change jobs or not is taken after comparing their disutility of work in the current firm with the wage premium they receive. Since both of these factors are independent of the job assignment, the results regarding turnover in the proposition holds for different job levels in which the worker might be assigned throughout their career.

Next we derive some results on wages and promotion dynamics inside firms, which comes out of Proposition 2. The empirical results that are documented in this section are from Baker et al. (1994a,b).

4.3. Results

Before elucidating the results on wage and promotion dynamics it is worth-mentioning that all of the previous corollaries from the model of the previous section (namely, Corollaries 1.1, 1.2, and 1.3) hold in this model with job levels.¹⁴ Thus in order to avoid expositional repetition, we

only derive the new results here which arise as a result of the introduction of symmetric learning with job levels.¹⁵

Corollary 2.1. *Serial correlation in wages.*

More formally, if a worker is in job 1 in period 2, then the conditional expectation of $(w_{i3}-w_{i2})$ is an increasing function of $(w_{i2}-w_{i1})$. From Proposition 2 we can see that the wage each period consists of two components. The first component is a function of the expected innate ability and the second component is a fixed term, which is dependent on the period of the workers' career. Thus, starting at a given experience level and wage, a large wage increase indicates that an increase in expected innate ability occurred. The result follows because in the symmetric learning case wage increases are an increasing function of expected innate ability.

Corollary 2.2. *Probability of promotion increases in wages.*

Formally, let the probability of getting promoted to a higher job level by a worker who is currently paid a wage w be denoted by $\pi(w)$. Then $\pi(w)$ is increasing in w . The promotion aspect is governed by the observation of z_{it} which the firm observes and which in turn determines the expected innate ability of the worker. The logic behind the result is as follows: given past performance, there exists a critical value of current performance above which promotion occurs. And this critical value of current performance above which promotion occurs is lower if past performance has produced a more optimistic belief about innate ability. A wage increase in the past is an indicator about high innate ability in an expected sense and thus the probability of promotion increases in w .

Corollary 2.3. *Higher level jobs are filled by candidates from the internal pool of workers as opposed to the outside market. (Baker et al., 1994b).*

This result is a consequence of the fact that the workers climb up the job ladder during their careers because of the general human capital accumulation. During the end of their career they are typically at the higher job level and since we have proved that in equilibrium the likelihood of turnover decreases as the career proceeds, those jobs are in most cases filled from workers in the internal workforce.¹⁶ Thus there is a higher intake of workers from the external market at the lower job levels.

Hence in this section, we extended the previous analysis of the 3-period model to include job levels, and verified the results proved there in this extended set up and also proved some new results, which were stated in the corollaries.

5. Discussion

This section offers two discussions. First we show how we can introduce worker heterogeneity in intrinsic mobility in the above model and its implications. Second, we discuss how we can derive certain empirical predictions from our model.

¹⁴ See Ghosh (2002) where this claim is proved in an n-period model with job levels.

¹⁵ Note that here even if $q=1$ we would get the results given below since the Corollaries 2.1 and 2.2 are independent of that. Nonetheless to maintain consistency with the previous model without job levels, I have kept $q < 1$.

¹⁶ Note that the fact that turnover decreases is a result of specific human capital also, as established in the two propositions.

5.1. Worker heterogeneity

Adding worker heterogeneity concerning the propensity to change jobs in our model complicates the analysis without yielding much in terms of deriving a richer set of facts. For that reason we discuss this informally here without formally incorporating this aspect into our model. Worker heterogeneity can be introduced in the sense that workers differ in their probability of having a new draw of their disutility of work in a particular firm each period. In the equilibrium that we derive in Proposition 2 the introduction of worker heterogeneity would affect the analysis in the following way. As was the case in the model without job levels, there is a cut-off disutility level each period above which turnover takes place, but in this case for each of the workers' type (assume that there are 'k' types of workers indexed by 'i') there is a separate cut-off level each period.¹⁷ The reason for this is because workers for whom the probability of realizing a new draw of disutility each period is higher (i.e. higher q_i 's) will be less reluctant to change jobs for a positive realization of the disutility in any given period as against a worker for whom the probability is lower. For a lower q_i the chances of enduring this disutility throughout their career is higher. Hence they are more likely to change jobs resulting in a lower cut-off level for the disutility of work. From this analysis we can further get the following result given below.

Corollary 2.4. *Mobility is strongly positively related to the frequency of job change prior to the start of the job (Farber, 1997).*

The basic intuition is that the subpopulation that changed jobs last period is composed of a higher fraction of workers with high turnover probabilities (since they are composed of the ones with low q_i 's following the intuition given above) than either the entire population or the subpopulation that did not change jobs last period. Therefore a worker with a history of prior turnover has a higher probability of subsequent turnover than a worker without such a history.

5.2. Empirical predictions

There are some empirical implications that follow from this model. First, we have assumed that the specific human capital accrues to the output of a worker in a simple additive way. It would be interesting to verify empirically, the way in which the specific human capital accrues to workers in different type of firms and then see the corresponding effect of turnover in those firms. Our model gives an avenue in which the specific human capital is related to turnover rates.

Secondly it would be interesting to study the relationship between firm culture and turnover rates. Firms with strong culture should have a broader distribution for D than those with weak culture. Our theoretical model gives a relationship between D and turnover rates. One might try to investigate whether it is good to have a strong culture or not, and whether it is indeed the case that firms with a strong work culture have lower turnover rates or not.

6. Conclusion

The objective of this paper is to take a step towards explaining the various facts about careers of workers in firms in the wake of new evidence which has been found out in some

¹⁷ Thus there are worker "heterogeneity groups" and we assume that firms know the type of worker they are dealing with before they announce a wage offer each period.

recent empirical work on turnover issues and wage and promotion dynamics in firms. There has been theoretical work which addresses some of the issues on wage and promotion dynamics. But the empirical studies clearly indicate that “the external market directly influences personnel policies throughout the firm by entry and exit” (Baker et al., 1994a). With this in mind this paper conducts an equilibrium analysis to show in a single theoretical model how the external market might influence the wage and promotion dynamics inside the firm. Some of the results from this paper cannot be explained without considering an integrated approach.¹⁸ At the same time we show that the existing results which exist by taking a non-integrated approach also hold in our model. We also discuss certain empirical implications of our study. The results derived in this paper are in accordance with maximization on both the workers’ and the employers’ side. Thus we get results from both the turnover literature and the wage and promotion dynamics literature.

Before we conclude, it is worth mentioning a couple of other issues related to our theoretical model. One of the most crucial components in our model is the concept of firm specific human capital. Though there has been substantial previous work, both theoretical and empirical, based on firm specific human capital there have been issues raised regarding the empirical validity of this concept. A concern regarding this issue is that there is hardly any human capital which is purely firm-specific. Another view is that ‘specific’ human capital more commonly consists of other forms¹⁹ which may range from industry specific (Neal 1995 and Parent 2000), occupation specific (Shaw, 1984) or task specific (Gibbons and Waldman, 2003). Though we acknowledge such concerns, in our context, we would argue that our results would go through as long as some part of the specific human capital is truly firm specific and cannot be appropriated in a new firm to which the worker might move.²⁰ Thus an empirical question that needs to be studied is how much of the specific human capital is truly firm-specific.

The next fruitful step towards a more complete equilibrium analysis will be to incorporate asymmetric learning in such a model. We can incorporate asymmetric learning in the sense that the employer learns about the worker productivity more than the market. Thus there will be an incentive to conceal this information from the market by the job assignment policy. By doing so, one can employ ideas associated with asymmetric learning which will help to address other empirical facts regarding promotion dynamics. For example, one conjecture is that this would allow us to address results regarding the career outcomes of new hires as against incumbents. It has been found that the career outcomes are more variable for new hires than for incumbents. More specifically, while the average level attainment is slightly higher for incumbents than for new hires, the variance of cohort level attainment is substantially greater for new hires. This difference could arise if the firm uses the incumbent employees’ tenure in Level 1 to learn about their abilities, and to screen out the least able and the ablest individuals. In contrast, the firm has less information about those hired from outside, and so screens them in their subsequent career moves.

In summary, this model suggests an avenue by which worker turnover issues can be introduced in a complete equilibrium analysis of careers in firms along with the results on wage and promotion dynamics which has been found in recent empirical studies.

¹⁸ For example Corollary 2.3 and Corollary 2.4 combines both strands to derive the required results.

¹⁹ See Helwege (1992) on the issue of separating different forms of human capital empirically.

²⁰ Levenson (2004) discusses how human capital can be truly firm-specific.

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Appendix A

Proof of Proposition 1. Let us first consider the wage offers at the beginning of Period 3. The outside wage offer is just X_{i3} because after Period 3 the career ends and so the specific human capital is not realized for firms who hire workers at the beginning of period 3. So they only bid for the current period productivity. Now consider the wage setting process for firm e. Workers have different disutilities of work. There will always be workers on the margin between staying and quitting. The higher the wage offered to workers of a given ability level the greater the number who prefer to stay rather than quit. The net utility which the worker i receives by staying in firm e is given by $U = W_{i3} - D_2$, while the net utility by changing firms is just X_{i3} . So by offering a wage W_{i3} the marginal worker who is indifferent between staying and quitting is one with $D_2 = W_{i3} - X_{i3}$. D^* denotes the maximum disutility level till the workers stay with firm e at period 2. In period 3, firm e knows that for $(1-q)$ of the workers the distribution is uniform from $[D_L, D^*]$ (since the disutility factor does not change after period 2 for this fraction). For the rest q fraction the distribution is $[D_L, D_H]$. The objective of the employer is to set a wage which maximizes the third period profits. The wage offer by firm e is thus given by the following maximization exercise

$$\text{Max}_{W_3} \left[q \int_{D_L}^{W_3 - X_{i3}} (X_{i3} + s - W_{i3}) \frac{1}{D_H - D_L} dD + (1-q) \int_{D_L}^{W_3 - W_{i3}} (X_{i3} + s - W_{i3}) \frac{1}{D^* - D_L} dD \right] \quad (1)$$

There can be three possible cases: $W_{i3}^* - X_{i3} < D^*$; $W_{i3}^* - X_{i3} = D^*$; $W_{i3}^* - X_{i3} > D^*$. We first assume that $W_{i3}^* - X_{i3} < D^*$. And then after deriving D^* with this assumption, we can check whether this holds. The analysis will be the same for the other two possible cases.

Since D is a draw from a uniform distribution, we can write (1) as follows:

$$\text{Max} \frac{q}{D_H - D_L} (W_{i3} - X_{i3} - D_L)(X_{i3} + s - W_{i3}) + \frac{(1-q)}{D^* - D_L} (W_{i3} - X_{i3} - D_L)(X_{i3} + s - W_{i3})$$

$$\text{Max} (W_{i3} - X_{i3} - D_L)(X_{i3} + s - W_{i3}) \left[\frac{q}{D_H - D_L} + \frac{(1-q)}{D^* - D_L} \right]$$

From FOC we get

$$W_{i3}^* - X_{i3} = \frac{s + D_L}{2}$$

Hence the difference of wages at the beginning of period 3 is $(s + D_L)/2$ and the wage is given by $W_{i3}^* = X_{i3} + (s + D_L)/2$. It is straight forward to check by substituting W_{i3}^* in the profit expression that the expected profits in period 3 is given by

$$\pi_3 = \left(\left(\frac{s - D_L}{2} \right)^2 \right) \left[\frac{q}{D_H - D_L} + \frac{(1-q)}{D^* - D_L} \right]$$

Now we have to check that for the 2nd period whether D^* is greater than, less than or equal to $(s + D_L)/2$. But before that we should note certain things. The firm e and the market take into

account their earnings over the entire period from a worker when deciding on the wage to be offered to the worker. And also the worker considers its utility over his career when taking up an offer. Thus he takes into account that there is a probability of him changing firms in period 3. This fact is incorporated while we calculate the utility expressions below. At the beginning of period 2, the worker knows exactly the disutility level D from working in firm e (because he had worked for firm e in period 1). Denoting by \bar{U} , the utility which the worker receives by sticking to firm e this period is $\bar{U} = (W_{i2} - D) + F\left(\frac{s+D_L}{2}\right) [q(X_{i3} + \frac{s+D_L}{2}) + (1-q)(X_{i3} + \frac{s+D_L}{2} - D)] + (1-F\left(\frac{s+D_L}{2}\right))X_{i3}$. If he changes the current employer then he receives $U = X_{i2} + \pi_3 + F\left(\frac{s+D_L}{2}\right) [(X_{i3} + \frac{s+D_L}{2})] + (1-F\left(\frac{s+D_L}{2}\right))X_{i3}$. Note that in this case the term π_3 is the future profits received by the outside employer from period 3 which is reflected in the higher wages offered to the workers this period. From the above expressions it is easy to verify the cut-off D worker who will stay with the firm e in period 2 for a given W_{i2} . After some algebra it comes to $\frac{W_{i2} - (X_{i2} + \pi_3)}{(1 + F\left(\frac{s+D_L}{2}\right)(1-q))} = \phi$.

Thus the relevant maximization exercise for firm e at the beginning of period 2 is given by:

$$\text{Max}_{W_{i2}} \left[\int_{D_L}^{\phi} [(X_{i2} + s - W_{i2}) + (\pi_3)] \frac{1}{D_H - D_L} dD \right]$$

Where the first term in the maximand is the profit from period 2 and the second term is the expected profit from period 3.

$$\text{The FOC gives us } W_{i2}^* - X_{i2} - \pi_3 = \frac{s + D_L(1 + F\left(\frac{s+D_L}{2}\right)(1-q))}{2}$$

Therefore we get that $W_{i3}^* - X_{i3} = \frac{s + D_L}{2} < W_{i2}^* - X_{i2} - \frac{s + D_L(1 + F\left(\frac{s+D_L}{2}\right)(1-q))}{2} = W_{i2}^* - \bar{W}_{i2}$ where \bar{W}_{i2} is the outside offer (and recall that $\bar{W}_{i3} = X_{i3}$). The cut-off disutility level during period 3 is given by $\bar{D} = \frac{s + D_L}{2}$. And the cut-off disutility level during period 2 which is Φ is given by

$$D^* = \frac{s + D_L(1 + F\left(\frac{s+D_L}{2}\right)(1-q))}{2} / \left(1 + F\left(\frac{s + D_L}{2}\right)(1-q)\right)$$

We know from prior result that $W_{i3}^* - X_{i3} = (s + D_L)/2$. Hence our original assumption that $W_{i3}^* - X_{i3} < D^*$ is not true. So the maximand (1) now becomes

$$\text{Max}_{W_3} q \int_{D_L}^{W_{i3} - X_{i3}} (X_{i3} + s - W_{i3}) \frac{1}{D_H - D_L} dD$$

After doing the same exercise with the new maximand, we now get the profit expression

$$\pi_3 = \left(\left(\frac{s - D_L}{2} \right)^2 \right) \left[\frac{q}{D_H - D_L} \right]$$

The turnover comes from the fraction who have their realization of the random disutility above the cut-off. We have to compare the cut-off disutility level in both periods to compare the turnover levels. The distribution function of D is given by $F(\cdot)$. We know the cut-off disutility level for period 2 is given by $\left(\frac{W_{i2}^* - \bar{W}_{i2}}{(1 + F\left(\frac{s+D_L}{2}\right)(1-q))} \right)$ and the cut-off disutility level for period 3 is given by $(W_{i3}^* - X_{i3})$. Therefore the probability of turnover after period 1 = $\left\{ 1 - F\left(\frac{W_{i2}^* - \bar{W}_{i2}}{(1 + F\left(\frac{s+D_L}{2}\right)(1-q))} \right) \right\} = P_2$ And after period 2 the probability of turnover = $\{1 - F(W_3^* - X_{i3})\} = P_3$ QED.

Proof of Corollary 1.1. We have already proved that $W_{i3}^*-X_{i3}=(s+D_L)/2$. And also $\left(\frac{W_{i2}^*-\bar{W}_{i2}}{(1+F\frac{(s+D_L)}{2})(1-q)}\right) = \frac{s+D_L(1+F\frac{(s+D_L)}{2})(1-q)}{2} / (1+F\frac{(s+D_L)}{2})(1-q)$ To show that $P_3 < P_2$ we have to show that $W_{i3}^*-X_{i3} > \left(\frac{W_{i2}^*-\bar{W}_{i2}}{(1+F\frac{(s+D_L)}{2})(1-q)}\right)$. (See the way P_3 and P_2 have been defined above) We can get this by just comparing the two expressions for the above two terms respectively.QED

Proof of Corollary 1.2. The difference in wages at period 2 and period 3 is $(W_{i3}^*-X_{i3})$ and $(W_{i2}^*-\bar{W}_{i2})$ respectively. We know from the Proof of Proposition 1 that $W_{i3}^*-X_{i3}=(s+D_L)/2$ and $(W_{i2}^*-\bar{W}_{i2}) = \frac{s+D_L(1+F\frac{(s+D_L)}{2})(1-q)}{2}$. Hence by a direct comparison we get that $W_{i2}^*-\bar{W}_{i2} > W_{i3}^*-X_{i3}$.QED.

Proof of Corollary 1.3. First let us calculate π_3 , i.e. the profits which the market firm gets if she is able to hire the worker successfully in period 2. The output of the worker in period 3 will be $X_{i2}+s$. Note that it will be the second period that the worker will work in the new firm and hence the specific human capital is added to the output. The wage that is paid to the worker is $X_{i2}+(s+D_L)/2$. Since the expected D for the worker in the new firm is zero (the disutility realization is independent in each firm), we know that the worker will stay in the new firm in period 3 also. Thus the profits π_3 will be $(X_{i2}+s)-(X_{i2}+(s+D_L)/2)=(s+D_L)/2$.

To ensure that the gain in output due to the general human capital of the worker in firm e from period 2 to period 3 is more than π_3 , we impose the following condition: $X_{i3}-X_{i2} > (s-D_L)/2$. The above condition simplifies to $s < \bar{s}$ where $\bar{s} = 2(X_{i3}-X_{i2})+D_L$.QED.

Proof of Proposition 2. In this case, because learning is symmetric, competition among firms each period yields efficient job assignment. Given this we compute a worker’s expected effective ability, η_{ii}^e , and then the worker’s expected output in job j as $Ey_j = d_j + c_j\eta_{ii}^e$. Note, that the linearity of the production function is key here: without linearity, expected output would not equal the output of a worker known to have on-the-job human capital equal to η_{ii}^e . Another crucial thing to note is that the premium given to the workers in period 2 and 3 (as reflected in the wage equations) is independent of the job level to which the worker is assigned. We now have that, given efficient job assignment, if $\eta_{ii}^e < \eta^l$ ($\eta^l < \eta_{ii}^e < \eta''$) then the worker i is assigned to job 1 (job 2) in period 2 and $W_{i12}^* = d_1 + c_1\eta_{i2}^e + \pi_3 + \left(\frac{s+D_L(1+F\frac{(s+D_L)}{2})(1-q)}{2}\right) \left(W_{i22}^* = d_2 + c_2\eta_{i2}^e + \pi_3 + \left(\frac{s+D_L(1+F\frac{(s+D_L)}{2})(1-q)}{2}\right)\right)$. Given that the premium to the workers in period 2 and 3 is independent of the job level to which the worker is assigned, the derivation of the above wage equations (similarly for period 3) is same as in proposition 1. Hence the probability of turnover as given in the proposition is independent of the job level but only the period of the worker’s career matters as in the previous section without job levels.QED.

Proof of Corollary 2.1. The corollary assumes that the worker is in job 1 for periods 1 and 2. Also according to the assumptions in the proposition, $\theta_L f(2) > \eta^l$, i.e. promotion is guaranteed in period 3. We use the following notation: $p = \text{Prob}(\theta = \theta_H/z^0)$ and $q = \text{Prob}(\theta = \theta_H/z^0, z_{i1})$, so that $w_{i1} = d_1 + c_1[p\theta_H + (1-p)\theta_L]f(0)$ and $w_{i2} = d_1 + c_1[q\theta_H + (1-q)\theta_L]f(1) + \pi_3 + \left(\frac{s+D_L(1+F\frac{(s+D_L)}{2})(1-q)}{2}\right)$. Thus, $w_{i2}-w_{i1} = c_1[q\theta_H + (1-q)\theta_L]f(1) + \pi_3 + \left(\frac{s+D_L(1+F\frac{(s+D_L)}{2})(1-q)}{2}\right) - c_1[p\theta_H + (1-p)\theta_L]f(0)$, increases in q given p. Therefore, we show that $E(w_{i2}-w_{i1}/p, q)$ increases in q given p, and hence that $E(w_{i3}-w_{i2}/w_{i1}, w_{i2}-w_{i1})$ increases in $w_{i2}-w_{i1}$ given w_{i1} . To express the wage w_{i3} , we introduce another notation: $r = \text{Prob}(\theta = \theta_H/z^0, z_{i1}, z_{i2})$.

In period 3 the worker is in job 2, so $w_{i3} = d_2 + c_2[q\theta_H + (1-q)\theta_L]f(2) + \frac{s+D_L}{2}$. But beliefs are a martingale, so $E(\tau/q) = q$. The workers expected increase in effective ability from period 2 to period 3 is $[q\theta_H + (1-q)\theta_L][f(2)-f(1)]$, but part of this increase is rewarded at c_1 and the rest at the higher slope c_2 . Both of these effects favor workers with larger wage increases: the expected increase in effective ability is larger, and more of this is valued at the higher slope c_2 . To see this, we know that $\eta_{i2}^e < \eta'$ because the worker remains in job 1 for period 2. Thus the expected increase in effective ability from period 2 to period 3 can be written as all being rewarded at slope c_2 except for the part from η_{i2}^e up to η' , which is rewarded at c_1 . That is, $E(w_{i3}-w_{i2}/w_{i1}, w_{i2}-w_{i1}) = c_2[q\theta_H + (1-q)\theta_L][f(2)-f(1)] - (c_2-c_1)(\eta' - \eta_{i2}^e) + \frac{s+D_L}{2} - \pi_3 - \left(\frac{s+D_L(1+F(\frac{s+D_L}{2})(1-q))}{2}\right)$. The first term increases in q and $(\eta' - \eta_{i2}^e)$ and $\left(\frac{s+D_L(1+F(\frac{s+D_L}{2})(1-q))}{2}\right)$ decreases in q . QED.

Proof of Corollary 2.2. For simplicity we consider workers in job 1, analogous arguments apply to job 2. Because the worker is assigned to job 1 in period 1, we know that $[p\theta_H + (1-p)\theta_L]f(0) < \eta'$. To be promoted to job 2 for period 2, the worker's performance in period 1 must be sufficiently high that $q \geq p^*$, where $[p^*\theta_H + (1-p^*)\theta_L]f(1) = \eta'$. That is, given p , z_{i1} must satisfy, $\frac{p}{p+(1-p)h[z_{i1}-\theta_L]f(1)}/h[z_{i1}-\theta_H]f(1)} \geq p^*$. Now, $h[z_{i1}-\theta_L]f(1)/h[z_{i1}-\theta_H]f(1)$ is monotonically decreasing in z_{i1} . Hence given p , there exists a critical value $z_{i1}^*(p)$ such that the above inequality holds if and only if $z_{i1} \geq z_{i1}^*(p)$. From the above inequality, $z_{i1}^*(p)$ decreases with p . That is, the critical value of past performance above which promotion occurs is lower if past performance has produced a more optimistic belief about innate ability. Because the critical value decreases in p and the probability that z_{i1} exceeds an arbitrary value increases in p , the probability of promotion increases in p .

The prior belief p is monotonically related to the current wage: $w_{i1} = d_1 + c_1[p\theta_H + (1-p)\theta_L]f(0)$. Thus, the critical value of z_{i1} such that promotion occurs decreases in w . For both these reasons, the probability of promotion increases in w . QED.

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